THE UNIVERSITY OF TEXAS BULLETIN

No. 3043: November 15, 1930

THE TEXAS MATHEMATICS TEACHERS' BULLETIN

Volume XV, Number 1

PUBLISHED BY
THE UNIVERSITY OF TEXAS
AUSTIN

Publications of The University of Texas

Publications Committees:

GENERAL:

FREDERIC DUNCALF	MRS. F. A. PERRY
J. F. DOBIE	C. H. SLOVER
J. L. HENDERSON	G. W. STUMBERG
H. J. MULLER	A. P. WINSTON

OFFICIAL:

E. J. MATHEWS	KILLIS CAMPBELL
C. F. ARROWOOD	C. D. SIMMONS
E. C. H. BANTEL	BRYANT SMITH

The University publishes bulletins four times a month, so numbered that the first two digits of the number show the year of issue and the last two the position in the yearly series. (For example, No. 3001 is the first bulletin of the year 1930.) These bulletins comprise the official publications of the University, publications on humanistic and scientific subjects, and bulletins issued from time to time by various divisions of the University. The following bureaus and divisions distribute bulletins issued by them; communications concerning bulletins in these fields should be addressed to The University of Texas, Austin, Texas, care of the bureau or division issuing the bulletin: Bureau of Business Research, Bureau of Economic Geology, Bureau of Engineering Research, Interscholastic League Bureau, and Division of Extension. Communications concerning all other publications of the University should be addressed to University Publications, The University of Texas, Austin.

Additional copies of this publication may be procured from the University Publications, The University of Texas, Austin, Texas

THE UNIVERSITY OF TEXAS PRESS, AUSTIN

THE UNIVERSITY OF TEXAS BULLETIN

No. 3043: November 15, 1930

THE TEXAS MATHEMATICS TEACHERS' BULLETIN

Volume XV, Number 1

PUBLISHED BY THE UNIVERSITY FOUR TIMES A MONTH, AND ENTERED AS SECOND-CLASS MATTER AT THE POSTOFFICE AT AUSTIN, TEXAS, UNDER THE ACT OF AUGUST 24, 1912

The benefits of education and of useful knowledge, generally diffused through a community, are essential to the preservation of a free government.

Sam Houston

Cultivated mind is the guardian genius of Democracy, and while guided and controlled by virture, the noblest attribute of man. It is the only dictator that freemen acknowledge, and the only security which freemen desire.

Mirabeau B. Lamar

THE UNIVERSITY OF TEXAS BULLETIN

No. 3043: November 15, 1930

THE TEXAS MATHEMATICS TEACHERS' BULLETIN

Volume XV, Number 1

Edited by

MARY E. DECHERD
Adjunct Professor of Pure Mathematics

Teachers of mathematics in Texas are cordially invited to use this bulletin for the expression of their views. The editor assumes no responsibility for statements of facts or opinions in the articles.

TABLE OF CONTENTS

HORIZONTAL LINE GRAPHS

DR. H. J. ETTLINGER
The University of Texas

The first-class postage law which determines the number of two-cent stamps to be placed on a letter is known to every high-school student; the rule reads as follows: *The postage is two cents for every ounce or fraction thereof.*

If we let p represent the *number* of *cents* of *postage* and w the *number* of *ounces* of *weight,* then the equation or formula which represents the first-class postage law is:

If w is a plus whole number, $p = 2w$.

If w is not a plus whole number, $p = 2W$, where W is the first plus whole number greater than w.

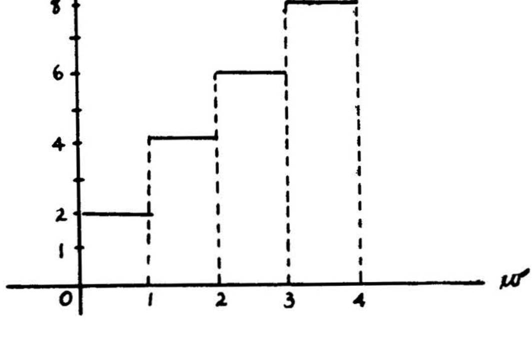
A table of values would read somewhat as follows:

w	1/4	1/3	5/6	1	1 1/2	1 3/4	2	2 1/8	2 1/6	2 2/5	2 4/5	3
p	2	2	2	2	4	4	4	6	6	6	6	6

This would not be a complete table of values, but represents some weights which might be met with by an individual in his correspondence or, certainly, by a postal clerk.

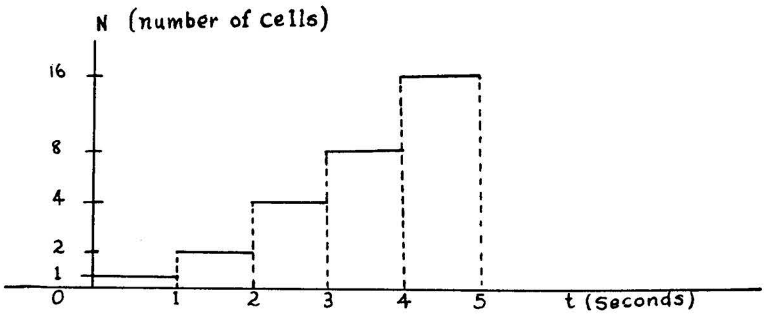
In high-school algebra textbooks of recent vintage, bar graphs of population, cotton production, and other statistics are given. The postage law above is an exceedingly interesting type of bar graph, which, in fact, bridges a gap between bar graphs and the more standard graphs on coördinate paper.

The graph of this postage law consists of pieces of horizontal lines as follows:



From $w = 0$ up to and including $w = 1$, p is equal to 2, which is represented by a horizontal line segment two units above the w-axis. From $w = 1$ up to and including $w = 2$, p is equal to 4, which is represented by a horizontal line segment four units above the w-axis, or two steps higher than the first. The complete postage law then would be represented by the horizontal pieces of a flight of steps each two units above the one before it.

Another example of this same kind of law is the fundamental fact of biology that a living cell divides into two cells, these two each divide into two others, and so on. If we assume that this process takes place at the end of each second, and we take account of the number of cells as compared with the number of seconds, we obtain the following: the number of cells during the first second is 1, the number of cells during the next second is 2, the number of cells during the third second is 4, the number of cells during the fourth second is 8, and so on. This biological law gives a horizontal bar graph as follows:



These graphs are illustrations of the kind in which we have pieces of horizontal lines. The National Committee on Mathematical Requirements recommended in its report that high-school students be given simple exercises in counting squares "under" a given graph. In the case of a graph like the two illustrations given this is a simple matter, since there are a whole number of squares "under" the curve. For any other graph, such as a circle, we may obtain an "approximate" answer by replacing the given graph by one which consists of short pieces of horizontal lines; in each case we may determine the distance from the horizontal axis up to the piece by taking the ordinates at equally spaced points, following the pattern of the postage law and the biological law.

PEDRO DE PAZ: ARITHMETIC

C. E. CASTANEDA
Latin-American Librarian, The University of Texas

While in Mexico this spring it was the writer's privilege to chance upon a copy of Pedro de Paz' Arithmetic of which a brief mention was made in the previous issue of this bulletin while discussing "The Earliest Arithmetics in America." The rare little book is entitled *Arte para aprender todo el menor del Arithmetica, sin Maestro. Dirigido al Doctor Don Diego de Guevara y Estrada, Chantre de la Sancta Iglesia Metropolitana de Mexico. Hecho por Pedro Paz Contador de la dicha sancta Iglesia. Con privilegio. Impreso en Mexico por Ioan Ruyz. Año de 1623.* It is octavo size and in perfect state of preservation. The owner of this precious textbook, of which only one other copy is known to exist, is Señor Gomez de Orozco, of Tizapam, Mexico.

The book consists of 181 folios of text besides 7 preliminary leaves in which appear the license of the viceroy to print, dated March 24, 1623, and a prologue in verse and another in prose. The work comprises the four simple rules of addition, subtraction, multiplication, and division, a surprisingly full treatment of fractions, a discussion of proportion and the uses and applications of the rule of three simple and complex, and simple algebraic equations. It is truly a remarkably comprehensive arithmetic for the time of publication. A good idea of the work and of the preparation of the author for writing it as well as the reasons for its publication are given in the prologue in prose which we translate herewith.

> When I determined to begin this work (curious reader) it was my intention to write a book that should contain not only the whole of the art of *Arithmetica Menor y Mayor* in its entirety but such short cuts and simplified methods as I had discovered or worked out, arranged in such a manner as to improve each and every one of them as best I could, with the help of my dull understanding. (If this art should

repay me for the time I have given it, which has been my whole life from the time of my childhood, it would have to be most liberal, considering the affection with which I have cherished it). When I considered, however (supposing the book finished) that certain characters were required [for its printing] which because of their frequent use could not be employed in long examples in a more technical book, I checked myself and chose the means most adaptable to bring it out as the brief work [I present now]. It seems to me that in this form I have succeeded in the aims I desired. For, what is more needed in the state than a book whereby one may learn without a teacher the necessary rules for dealing with whole or fractional ciphers, to carry on all manner of transactions, without the exception of a single one, and without omitting a thing of importance? From this I infer the need of a simple book of rules and common characters, printed at whatever the cost, for its usefulness will repay for the effort. It is specially fit for all states, classes, ages, trades, and business, all of whom lack the means to learn this without a teacher, for all those that desire it do not have the means to afford it. Those that do will find it less burdensome by the use of this *Arte*, being free to use the second book in which the *Arte Mayor* will be explained after they have mastered the present one, the second being now in preparation. This will be printed with the best characters used to this day, making everything clear and easy to learn. It will contain many rules of geometry (for the solution of which the simple operations are essential) and for this reason I recommend that the present volume be mastered first, because without it it will be impossible to understand the other. For this reason it was highly advisable and proper to publish this first that it may, through its shortcomings, reveal to me my deficiencies and point the things in which I must exercise greater care in the next, which I promise to prepare in like manner, God willing, all for His better service and your own.

Thus it is evident that de Paz intended to publish a second volume of his *Arte de Arithmetica Mayor,* but if he ever published it, no copy of it is known. As stated before the book is remarkably full in its treatment of not only the simple rules but of fractions and simple equations. The *Arithmetica* of Pasamonte published in 1649 is better explained in the light of the thoroughness of the work of de Paz. It is not without an antecedent. The work of his predecessor gives an insight into the knowledge with regard to arithmetic that was prevalent in Mexico in the first half of the Seventeenth Century.

MAKING BUSINESS ARITHMETIC INTERESTING

LAURA WENDT
Graduate Student in The University of Texas

Teachers of mathematics in high schools are frequently confronted with the problem of justifying the place of mathematics in a student's course of study. If the student is struggling along with the square on the hypotenuse or with two pipes filling a cistern while a third is emptying it, it is difficult to make him see the practical value of the subject. The teacher of business arithmetic, however, should find a ready answer to such questions.

The teacher of business arithmetic can and must sell his subject to the class before he can hope to teach it. He must create and maintain interest in it and a real desire on the part of the student to master it. Much of the success of the subject depends upon the teacher. He must be vitally interested and wide awake; his personality and interest must be felt. He must win the confidence of the class. No text is capable of telling a teacher how to conduct a class or of making the subject of real lively interest; the individuality and personality of the teacher must put the work over. Attitudes are contagious; the teacher's attitude toward the subject will very soon become the attitude of the class toward it.

There are many devices which a teacher can use to create interest. The students should be asked to consider themselves business men and women trying to arrive at certain conclusions. The business man is daily confronted with problems that he must solve. These do not come to him labeled and classified; but he must classify, arrange, and arrive at conclusions. In doing this he uses common sense, and the students, picturing themselves as business men, must apply this same common sense to their problems. As business men depend largely upon bookkeeping for the operation of their business, these future business men must learn bookkeeping, which is really only applied business arithmetic.

As business men, the students have in mind the mastery of the language of business, which is the language of figures, the most nearly universal of all languages. To manipulate this language with accuracy and speed must be their purpose. In the vocabulary the teacher should include such terms as Trade Discount, Interest, Net Proceeds, Assets, Liabilities, Inventory, and many others that must be thoroughly understood.

Good habits of keen observation and careful comparison must be formed at the beginning. The teacher should ask the pupils or have them ask each other as many mathematical facts as they can about the room in which the class meets, its length, height, width, floor space, window space, number of desks, and such questions. By careful questioning on his part, the teacher can develop in the class what may be called a mathematical method of observation, the ability to observe, to count objects and measure quantities, and in this way "tie up" the subject with everyday activities. Along with the observation habit should come the habit of making comparisons. The students can compare the length and width of the room or the size of the doors and windows. These comparisons may be expressed in simple terms at first, as twice as long, one-half as wide, and then in terms of decimals, percentage, or ratio, as the teacher may see fit. In this way the student begins to think in terms of arithmetic. If the student can observe and gather the data and state or construct problems of his own, a foundation is laid for real development later in the course.

After these foundations have been laid, work should begin in earnest. The same plan of work should not be used every day, for it will become a routine and the students will lose interest. Since the four fundamental operations must be mastered, these can furnish the first part of the class work a number of times. Short, snappy drills or rapid calculation should be given for the first few minutes of each recitation, with enough variations in the drills to keep them interesting. One day the forty-five combinations in addition can be used and the record kept, another day drill on multiplication can be given. Problems such as this never fail to keep the class alert:

$$3 \times 4 - 2 \div 5 \times 7 + 3 = ?$$

These problems should be given slowly at first, with increasing speed and a greater number of terms later. It is fine drill for the student to give the class such a problem. He must think clearly and quickly and his progress will please him greatly.

If drill in addition is needed, such a device as the following may be used:

$$\begin{array}{ccccccc} & & & 4\ 7 & & & \\ & & 8 & & 5 & & \\ 1 & & & & & & 2 \\ & & 9 & & 3 & & \\ & & & 6 & & & \end{array}$$

The teacher points to the numbers in rapid succession and calls for the sum of the group pointed out. This may also be used for multiplication.

The drill might also be on the writing of figures. Students should be able to write at least 150 figures per minute, and drill on this is interesting. As the speed increases the figures should not lose their legibility, so drill is really necessary. Then there may be drill on calling numbers rapidly and distinctly. A number of standard tests, such as the Curtis tests, can be obtained and may be used for drill purposes. Drill and competition will keep the students alert, with friendly competition against themselves, their classmates, or even the teacher.

After the four fundamentals have been studied the teacher is confronted with the problem of fractions, common and decimal, with their applications. Fractions can easily be justified. The students can think of various transactions which involve fractions; in fact they can hardly think of one that does not. Farmers seldom sell chickens that weigh an exact number of pounds, mothers do not always buy an exact number of yards of material, butchers cut meat in pieces which involve fractions. If the problems are practical, the work will be more interesting. Auto trips, baseball percentage scores, and such topics usually interest the students.

Interest, an application of percentage, can be made more realistic if approached properly. All students understand the matter of renting a house, and if interest is discussed as a matter of rent on money, the student can understand it. The sixty-day 6 per cent method which the banks use will prove interesting because of the time saved when using it. The study of compound interest tables, drafts, notes, interest on bank deposits, are all subjects of practical use. Bankers should be consulted as to forms and methods used. Students should be assigned to interview various bankers on these subjects. The bankers are glad to discuss them and give the students blank forms of notes and negotiable papers with which they should be familiar.

The same method can be used for commission and brokerage. If the problems can be related to the local business organizations, the students can see that they are really learning something to be of use later. The students can discuss the advantages and disadvantages of a man's working on a commission basis, both to him and his employer. In the same way, stocks and bonds can be made real. Students can secure stock certificate forms and bonds from banks or homes to study and compare. They can discuss the stock market, transfer of bonds, government bonds, and other realistic material. It is surprising how little some students know of such things as the stock exchange and how interested they become in these things if other students discuss them or report on them.

There is another group of problems which are in themselves interesting. These are the problems of the merchant and manufacturer. Local merchants can be interviewed on such questions as trade and cash discounts received on invoices or allowed to customers, turn-over, figuring profits on cost or sales, marking of goods, seasonal variations in the demand for articles, and other questions equally as interesting. The manufacturing industries of the town can be discussed and visited. The problem of making a payroll can be made interesting if the students can see one or talk to the paymaster or clerk about it. The time-clock is another interesting device used by merchants and manufacturers, and students are interested in seeing it used. The superintendent of a factory can give them interesting facts about raw materials, cost of production, and distribution of various overhead costs. Then when these problems come up in the text, the students know they are really being solved in home factories.

Another topic the value of which can be easily shown is that of graphs. Students are usually interested in pictures. Graphs are pictures designed to transfer readily to the vision certain facts. Before beginning the construction of graphs, the students should study all forms in general and collect typical ones in magazines or papers. It is worth mentioning that the president of the American Telephone and Telegraph Company, one of the largest corporations in the world, was first brought to the notice of the executives because he was asked to prepare certain data and he placed the data in graphic form. He was just out of college and it was at a time when graphs were coming into prominence. From that time on his progress was very rapid, and he is now president of this large corporation because he saw fit to place certain data before his superiors in graphic form and thereby won recognition. With this as a background the students will see the purpose of solving some of the problems.

There are two more types of problems which can be made of great interest to students, insurance and taxation. The modern business world could not operate without insurance protection, and if the teacher can develop in the class an appreciation of the real value of insurance to both the individual and the business man, the time will have been well spent. Divisions of the subject, such as rates and risks, rates and ages, rates on wooden structures as compared with brick or concrete structures, and rates in the country and city, can be assigned to various members for discussion with local insurance agents. Various kinds of policies should be brought in and discussed, not with respect to the technicality of insurance, but in order to have them visualize a policy and better understand the problems involved. In the same manner, taxation is a problem that touches almost every family. While one may not pay a direct tax of any kind, yet every one pays an indirect tax on some commodity that he purchases. While some of the problems in taxation are old, the income tax problem is always new. Students can secure income tax blanks from a local bank or adviser, and use these as a guide. Although the law changes in regard to rates, exemptions, and such questions, the basis is the same and the problem is of vital interest.

Other topics could be discussed, but the problem in each case is to relate the topic studied to the local conditions of the community. Teachers should always be searching for means to create interest in the topics, not only to the end of getting better results with the problems and in the class work, but also of increasing interest in relation to local conditions and proving to local business men and parents that he is not merely teaching arithmetic, but business practice as well.

VITALIZING ELEMENTS OF VARIATION IN A SOLID GEOMETRY COURSE

RUTH COTTINGHAM
Highland Park High School, Dallas, Texas

In recent years we have heard much about the combination of plane and solid geometry in a one-year course. Perhaps this is a good plan, but I shall vote against any such measure for the reason that a half-year devoted to solid geometry affords me the opportunity of presenting to a group of more or less mature students facts of general mathematical significance not included in other high-school courses. It is true that we do not need an entire half-year for the propositions within the covers of a solid geometry textbook. Time spent thus, in my opinion, is wasted. But it is my purpose here to state a few of the elements of variation that tend to make more vital both previous and subsequent knowledge of mathematics, thus broadening the horizon of thinking of the average student.

The first unit of the course provides for an abundance of analytical thinking. The student has probably received a certain amount of this type of training before he reaches the senior year, but it is my belief that nothing affords more opportunity for the enrichment of his thought processes than his contact with the problems of three-dimensional space. The magnitude and reality of his own world stagger him, and he realizes that he must find solutions to his problems, or "grope in the darkness." Generally speaking, we know that not until the primary stage of maturity has been reached does the student find himself capable of grasping the entirety of a situation. It is then that the game becomes fascinating and we find him eager to attack the most difficult problems. It is sometimes very amusing to observe his self-sufficiency. Most of us have laughed quietly at the serious attempts to unwind some time-worn mathematical complexity. Psychologically speaking, it would seem that the senior year of high school is the appointed time for intensified training in analytical thinking.

After the student becomes acquainted with the realms of three-dimensional space to some extent, a peep into a world of four dimensions usually stimulates his interest and imagination. Just here I may say that I have found quite valuable a five or ten minute discussion on present-day scientific discoveries and their relations to mathematics. You may think the students know nothing worthy of discussion in connection with modern science, but I dare say you will find them surprisingly intelligent. Ask them a few leading questions such as: "Who is Einstein?" "Have you read anything recently of general mathematical interest?" If you get no response, ask the same questions tomorrow. I can't guarantee what your luck will be, but mine has been quite gratifying. I must confess, however, that the teacher who undertakes such a scheme is forced to increase continually her sources of information relating to the various fields. After these little roundtable discussions concerning current happenings of the scientific world it is well to introduce topics of mathematical history, including the outstanding discoverers and discoveries of previous times. After presentation of the series of topics I ask each student to select one and make a comprehensive written report at some distant date. I usually cite a few references and urge him to find many others. A complete bibliography must be included in the report. This serves as the traditional term paper. The list of topics is exceedingly flexible and if the student is particularly interested in some other phase of mathematical relationships, I am glad to allow him to report upon a subject of his own choosing. As a whole the class takes great pride in these reports, and frequently individuals ask my permission to prepare another one. A certain boy came to me a year after his credit on the course was in the permanent record and expressed a desire to add something to his paper. He said that he had found more material on his subject. A number of others have come to me telling of added information they have acquired relative to their topics. Surely some lasting values result from work of this nature.

Another little exercise that most of the pupils enjoy is the solution of a real problem that relates in some way to our own city or to some nearby field of interest. In this collection I have problems relating to concrete driveways, parking stations, silos, floating duck blind, White Rock Lake, concrete culverts, grain elevators, road dumps, water supply tanks, swimming pools, and numerous problems relating to neighboring oil fields.

Besides reviewing some of the outstanding geometric principles in the light of three dimensions, we have specific possibilities in connection with various solids. The class usually enjoys furnishing its own specimens in the form of buckets, ice cream cones, tennis balls, etc., so that the teacher is relieved of this task. And, too, they are much more real than those solids the teacher may happen to have in the supply cabinet. The cone is one of my "pets." We have so much fun slicing it so as to observe the conic sections. (Incidentally, we usually have to resort to imagination or the specially prepared cone, for the ice cream cone is almost too brittle.) Just here a glimpse at analytic geometry can do no harm. I see no reason why the class should not learn a few elementary facts concerning the circle, the ellipse, hyperbola, and parabola. The individuals of the class probably have more of a "mind-set" for this work now than they will have next year when they launch into freshman college mathematics amidst the whirl and newness of college life. Again, there is much to be said about the pyramid and its relation to the history of architecture; also the sphere, and its beautiful significance in the study of astronomy.

The last unit of this "variegated" course I hesitate to call vitalizing. The reader must decide that for me. I think, however, that you will agree with me that the last unit is probably the most essential. In our study of the specific solids we, of course, stumble over the formula again.

Just a little innocent play with formulas for areas and volumes is not amiss, I think. And then when we begin to substitute actual values in the formulas my seniors begin to be troubled. "Why should a perfectly good course be ruined by including 'mean' arithmetic solutions!" Nevertheless, a bit of review brushes away the cobwebs from the arithmetic, and the skies are blue once more. Just in this connection I find it quite valuable to include a few lessons on approximation, significant figures, degree of accuracy, and the slide rule.

Perhaps you will be ready to offer criticisms about the arrangement of this sort of course. It may not be pedagogically correct. My contentions are simply these: (1) Not having a program of general mathematics in the schools of Texas and some other states, we need a course in the senior high school within which to incorporate certain "facts and fancies" of mathematical significance. (2) My experience has been that all the mathematics courses in the senior high school except solid geometry require so much time for drills, tests, and measurements, if the work is to be effectively taught, that we do not have time for "extras." (3) Such a course certainly is in accord with certain phases of the philosophy of education, for there have been many evidences of the fact that the student's activities involved have led on to other activities which have provided in part for the continual adjustment of the individual in an ever-changing society.

A NOTE ON AN INTERESTING RELATION

O. S. HOLLABAUGH
Vickery High School

This interesting numerical relation grew out of a problem which I gave to my class in solid geometry. The problem was to find the diagonal of the school room with the three dimensions being given. The class solved the problem but suggested in a pleasant way that next time I should give one that would be a "perfect" square. I suggested that they find three numbers that would give a square. The next day they said that 3, 4, and 12 and 4, 5, and 20 would give a square. We observed that 12 and 20 were the products of these numbers. We tried other combinations and found that they would work the same way. We also noticed that the result was always one more than the product of the two numbers. We stated the truth in the following statement:

The sum of the squares of any two consecutive numbers plus the square of their product will always give a (perfect) square and the result will be one more than the product of the two numbers.

If x is one number and $x + 1$ the next number above, $x(x + 1)$ is the product.

Then $(x)^2 + (x + 1)^2 + [x(x + 1)]^2 =$

$$x^2 + x^2 + 2x + 1 + x^4 + 2x^3 + x^2 =$$

$$x^4 + 2x^3 + 3x^2 + 2x + 1 = (x^2 + x + 1)^2$$

ALGEBRA IN THE GEOMETRY COURSE

CIGELY GOFF
Austin High School

In the study of geometry there is much need for many of the principles which are taught in algebra. It is my purpose here to consider a few of the more important of these.

The equation that is found in geometry differs from the algebraic equation in that the terms represent geometric figures such as angles, lines, arcs, triangles, instead of algebraic symbols. This enlargement of the use of the equation to express an equality of geometric concepts, just as later it must be extended to the use of trigonometric functions, baffles the student at first. Sometimes in algebra the student has failed to recognize the real meaning and the equation has simply been a problem to be solved, to obtain a value for some unknown quantity. In geometry the concreteness of the quantities aids in clarifying the meaning of the equation. The axioms of addition, subtraction, multiplication, and division must be stated each time instead of being mechanically used as they too often are in algebra, even to the point where the student fails to know what he is doing and if asked why, can give no other reason than that that is the way to do it. The axiom of substitution or, as it is so often used, the equating of two magnitudes which are equal to the same magnitude, is very important and is frequently used in geometric proofs. All of these axioms are fundamental in equation work and a successful handling of the equation demands a clear understanding and an accurate use of them. Most of the remaining axioms deal with inequalities, a subject which is omitted from many of the newer texts of secondary algebra and hence comes as a new subject when met in geometry. Some teachers feel that it is better to leave the theorems of inequality until the close of the course in plane geometry; certainly its postponement until late, perhaps at the close of Book One, is desirable.

One form of the equation much used in geometry is the formula. Too much attention cannot be given it in algebra, for it is one of the practical phases of geometry and is used extensively in the study of the physical sciences. The formula is a literal equation, in which any letter may become the unknown. Great care should be taken that all formula work be done in good form, which should be the parallel form that is used in the proof of theorems; a reason should accompany each statement that is made in the solution of the formula. Another type of equation that occurs frequently in problem work in geometry is the quadratic. Mr. Breslich in his *Second-year Mathematics*[1] says: "No one could have written the tenth book of Euclid's Elements without a good knowledge of ways of solving quadratic equations. Since this tenth book contains most of Euclid's original work, it may safely be assumed that Euclid had this knowledge. He solved no quadratics algebraically, but he proved geometrical theorems that amounted to such solutions." The problems in rectangles, right triangles, trapezoids, areas of circles, and those based on the tangent-secant proposition may involve the use of the quadratic. Usually these can be solved by the factoring method, but occasionally an irrational number enters in, and then either the completion of the square or the formula method is necessary.

Two algebraic processes which are often used in geometry are those of raising to powers and of finding the roots of numbers. The solution of the formula for the Pythagorean theorem makes constant use of these principles. The squaring of a binomial is necessary in the proofs of the square on the side opposite an acute angle and the square on the side opposite an obtuse angle, in both of which the projection of a line-segment upon a given line occurs.

---

[1]Breslich, Ernst R., *Second-year Mathematics for Secondary Schools*, p. 130.

Associated with roots are those quantities the roots of which cannot be exactly found. Here again the Pythagorean theorem offers much drill. It is interesting historically to note that Pythagoras is credited with having given to the world its first conception of irrational numbers in his discovery of the proof for the theorem that bears his name, especially in its application to the isosceles right triangle. The equilateral triangle, the diagonal of the square, and the apothem of the polygon offer many problems in the simplification and squaring of radicals.

Factoring does not enter into geometrical proofs and problems to any great extent. The factoring out of the monomial occurs in the proofs of the Pythagorean theorem by the algebraic method, the area of the trapezoid, and the area of the polygon. The derivation of the formula for finding the area of a triangle in terms of its sides is perhaps the outstanding instance in which factoring is involved. Since the use of a formula should presuppose a knowledge of its derivation, the student has here a good opportunity to use the difference of the squares method of factoring; all of this proof requires much algebraic ability.

Perhaps the parts of geometry that offer the greatest difficulty are those concerned with ratio and proportion. Not only is this a difficult phase of geometry, but it is one of the most important, not alone because it is an essential part of geometry but because upon it are based the fundamental functions of trigonometry. Its importance in all mathematics has been recognized by mathematicians of all times. Mr. Breslich says:[2] "Proportion as applied to numbers is one of the oldest mathematical topics. In the oldest known mathematical writing, the book of Ahmes, written by an Egyptian scribe 1700 B.C., proportion is one of the important subjects. The principles of proportionality as applied to line-segments and to areas were first studied by the Greeks. Euclid's Elements (300 B.C.) devotes the fifth

---

[2]Breslich, Ernst R., *Second-year Mathematics for Secondary Schools*, pp. 97–98.

and a part of the sixth book to the doctrine of proportionality as applied to line-segments and areas. It was once the most practical part of all geometry and some of the most practical subjects and topics of mathematics are still based on it."

The proportional relation between numbers runs through all mathematics and the idea should not be a new one to the geometry student. His idea of ratio must extend from numbers in arithmetic to letters in algebra, and to lines, perimeters, triangles, polygons, and circles in geometry. Since a ratio is a fraction and a proportion is two equal fractions, all ratios and proportions should be written as such. The algebraic proofs for the theorems on inversion, alternation, addition, and subtraction may be given by the analytic method, which consists of assuming the conclusion as true and working to the hypothesis; then by reversing the process used in the analysis, the direct proof may be obtained. The idea of proportionality of line-segments may be introduced by actual measurement of the sides of two triangles known to be similar and a comparison made of the results obtained; or in the case of pi, the student finds it interesting to find the value by measuring the circumferences and diameters of circular objects and finding by comparison of these the existence of the ratio. An understanding of the truths is essential before one can proceed with the formal proofs.

It is an erroneous impression, but one prevalent among high-school students, that algebra and geometry are entirely independent subjects, that when one has completed the first he can say good-bye to it forever. Quite otherwise is the case: an adequate working knowledge of algebra is necessary in the study of geometry.

TEACHING 11A ALGEBRA

MARY E. DECHERD
The University of Texas

For a number of years we have heard much of the yawning chasm that lies between the last year of the high school and the first year of college. Personally, since my first teaching was done in a high school, I gave little credence to the existence of such a chasm. However, I have for several years felt that I would like to investigate the situation, to see just how far apart the senior high school and freshman college year really are.

This year, in pursuance of this idea, through the kindness of the Department of Mathematics of The University of Texas and of the Austin High School, I have taught one class one semester in the Austin High School in place of one of my usual University classes.

In the following pages will be found a detailed outline of the course that I gave in 11A algebra in the Austin High School, February-June of this year. I was given permission to plan the course as I saw fit. My desire was to cover in this avowedly review course the various points in arithmetic, algebra, and plane geometry which seem to me vitally necessary as a foundation for a first course in mathematics in college and yet which students generally fail to have mastered in their high-school course.

It did not seem to me wise, however, to depart entirely from the printed course of study. Two requests I made at once: first, that I be allowed to issue arithmetics to the students, since much of the faulty preparation in algebra is, I think, due to lack of arithmetic viewpoint; and secondly, that I be not expected to give any trigonometry, as I feel that the time had better be spent on the other three subjects and that the amount of trigonometry which can be given in such a course is too small to be of any value.

The most surprising discovery I made in my stay in high school was that so much time had to be put on the few processes I covered in algebra, although the greater part of the course had been given these students in the 9B grade, that I did not get to review geometry except as the "word problems" dealt with geometric situations.

An interesting phase of the experiment was my association with Miss Edna Von Rosenberg, whose class was given to me. She sat in the room almost continually, and only through consultation with her did I succeed in carrying my course to its conclusion. Her experience and good judgment were to me invaluable. Moreover, had she not kept a list of daily assignments, the task of reproducing the course would have been very difficult.

From the foregoing statements, it is obvious that there are many points at which the course I gave seems far from ideal. In other words, children had to be taught under existing circumstances of course, and many changes would be found necessary in more elementary grades before the ideal course could be given in 11A.

I gave a conference period from 8:00–8:20 o'clock before the class meeting from 8:30–9:30 o'clock. Almost daily written work was handed in. This work was as a rule inspected sufficiently to determine its general character, and if at all well done, it was checked in my grade book as handed in. If it was done very poorly, or not handed in, the student was asked to hand it in later. Moreover, a short quiz was given very frequently. The students were often asked to choose the assignment for the next day and for the quiz. The examples given in the quiz were at once worked out and explained in class. The next day the quiz papers, graded on the scale of 10, were returned to the students.. In addition the regular tests were given every three weeks.

The text in arithmetic was Anderson's *Arithmetic, Book III,* while both Wentworth's *New School Algebra* and Edgerton and Carpenter's *Second Course in Algebra* were almost continually used.

During the last few weeks of the course, I had a new reference book just off the press, *Algebra, for Junior and*

Senior High Schools, by Calhoun, White, Simpson. Chapters found convenient for reference were I, VIII, IX, X, XII, XIII, and XVI. Other books used for examples were Wells, and Rietz and Crathorne.

May I say in closing that my teaching in the high school was most enjoyable. Superintendent A. N. McCallum, Principal G. H. Wells, and Mr. L. M. Fertsch, head of the mathematics department in the Austin High School, showed me every courtesy and consideration possible, while my association with the numerous friends I have among the high school teachers was a very great pleasure. In every way my experience was pleasant and profitable.

OUTLINE OF 11A ALGEBRA

FEBRUARY 5

I. Review of arithmetic.
  (1) Class discussion.
    a. Multiplication tables.
    b. Squares to 35.
      Note: $(29)^2 = (30 - 1)^2$; $(31)^2 = (30 + 1)^2$.
    c. Cubes to 12.
    d. Higher powers of 2, 3, 4, 5.
      Stress "mental arithmetic."
  (2) Assignment: Master a, b, c, d above.

FEBRUARY 6

  (1) Class discussion.
    e. Multiplication of fractions.
      To shorten work, (a) never multiply;
      (b) always "cancel."
    f. Review of squares, applying the Pythagorean Theorem to sides of the right triangle.
      Find sets like $3^2 + 4^2 = 5^2$.
  (2) Assignment: Practice on e and f.

FEBRUARY 7

(1) Class discussion.

g. Addition of fractions.

Only numbers with the same name can be added. 2/3 and 1/9 do not have the same name. *Denominator* means namer. 6/9 and 1/9 can be added.

Integer, an *untouched* or *whole* number.

Fraction, a *broken* number.

h. The only operation that can be performed on a fraction is to multiply numerator and denominator by the same number. Use this principle in division, thus—

$$\frac{\frac{5}{7} \times 35}{\frac{6}{5} \times 35} = \frac{25}{42}$$

i. *Repetend,* meaning *must be* repeated. All fractions give repetends.

(2) Assignment: Practice on g, h, i. In connection with i, consider $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{1}{6},$ $\frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}, \frac{1}{7}, \frac{2}{7}, \ldots, \frac{1}{8}, \ldots, \frac{1}{9}, \ldots, \frac{1}{10}, \ldots, \frac{1}{11},$ $\ldots, \frac{1}{12}, \ldots, \frac{1}{13}. \ldots$

FEBRUARY 10

(1) Class discussion.

Review of all fundamental operations on fractions.

(2) Assignment: Anderson, p. 30:11; p. 31:25, 26; p. 32:30, 31, 34, 36, 38.

FEBRUARY 12

(1) Discussion on last assignment.

(2) Assignment: Anderson, p. 154, 23 orally. A twenty-minute quiz on p. 154, beginning with number 8. Time test.

II. Algebra.

FEBRUARY 10

(1) Class discussion on algebra.

(2) Assignment: List performances and definitions in Wentworth, pp. 23–74.

FEBRUARY 11

(1) Class discussion of above assignment.

(2) Assignment: Write a logical outline of these pp. 23–74.

FEBRUARY 13

(1) Class discussion on Chapter VI, Special Cases of Multiplication and Division.

(2) Assignment: Study Chapter VI, working five examples in each exercise and if time permits, work another five in each. Hand in all on p. 77.

FEBRUARY 14

(1) Class discussion on relation in detail between Chapters VI and VII.

(2) Assignment: Compare Chapters VI and VII. Hand in Ex. 37, pp. 87 and 88, showing common factor.

FEBRUARY 17

(1) Class discussion on difference of squares.

(2) Assignment: pp. 90–94.
Hand in p. 90:31, 35, 36; p. 93:10–19.

FEBRUARY 18

(1) Class discussion on pp. 90–94.

(2) Assignment: Examination on
   1. Cancellation in arithmetic.
   2. Algebra
      (1) Parentheses.
      (2) Division.
      (3) Chapter VI.
      (4) Factoring through p. 94.

FEBRUARY 19

(1) Three weeks' quiz

1. Factor $x^2 - 4 - 8x + 16$.
2. Simplify

$$2a - [5d + \{3c - (a + [2d - \overline{3c + 4a}])\}] - (3d - 7c)$$

3. Divide $2x^3 - 3x^2 + 7x - 3$ by $2x - 1$.
4. Simplify

$$\frac{7\frac{1}{2} \times 4\frac{1}{2} \times 1\frac{1}{4}}{3\frac{3}{4} \times 2\frac{1}{4} \times 5}$$

5. Factor $c^3d^3 - 343z^6$
6. Factor (a) $x^2 - 4b^2 - 9c^2 + 12bc$
   (b) $32c^5 + a^5$
   (c) $6a^3 - 30a^2b + 36ab^2$

(2) Assignment: pp. 97, 99, 101.
Again carefully examine pp. 75–94 and 95–97.
Hand in the last five on p. 93 and all on p. 94.

FEBRUARY 20

(1) Class discussion on last assignment.

(2) Assignment: Criticize Ex. 42; review pp. 93 and 94. Hand in p. 101:13–32, omitting 19, 20, 21.

FEBRUARY 21

(1) Discussion of quiz.

(2) Assignment: Study p. 99 and p. 105.
Hand in $32c^5 + a^{10} = (2c + a^2)$ (..................).
Hand in p. 99, last 10; and p. 105, last 10.

FEBRUARY 24

(1) Discussion $a^n \pm b^n$, n odd and even.

(2) Assignment: Hand in pp. 105–7:20, 23, 30, 35, 44, 46, 49, 56, 80, 113.

FEBRUARY 25

(1) Discussion of fractions.

(2) Assignment: Hand in pp. 105–7: 101, 103, 104, 63, 72, 58, 61, 62, 85.

Note p. 76 and rule 4, p. 78.
Reduce the various performances in fractions in your text to the few fundamental performances.

FEBRUARY 26

(1) Discussion of last assignment.
(2) Assignment: p. 123. Recall fundamental principle in arithmetic at this point. Note carefully discussion of signs, p. 126.
Study pp. 134–135, handing in 14, 19.
Study p. 137, handing in 9.
Study pp. 139–140, handing in 19, 20, 21, 23, 24, 26, 27.

FEBRUARY 27

(1) Class discussion on fractions in arithmetic and algebra.
Integer, Fraction.
(2) Assignment: p. 137. Hand in all exercises.

FEBRUARY 28

(1) Same as in 27(1), applying fundamental principle to p. 144.
(2) Assignment: Hand in p. 144.

MARCH 3

(1) Class discussion on fractions, p. 144.
(2) Assignment: Short quiz.
Hand in pp. 146–147:20, 14, 13, 8, 25, 16; p. 145: 21, 20, 19, 10; p. 137:4.

MARCH 4

Club day.

MARCH 5

(1) Quiz:

1. Factor $8x^3 + 27y^6$ by inspection.

2. Add $\dfrac{1}{x-y} + \dfrac{1}{y-x}$

3. Add $\dfrac{1}{(x-y)(x+y)} + \dfrac{1}{(y-x)(y+x)}$

4. Solve $a^2 - 6a - 7 = 0$

5. Reduce $\frac{6}{13}$ to a decimal (12 places) .461538461538....

(2) Quiz worked out and discussed.

(3) Assignment: p. 151, ex. 65; pp. 154–155, study method used in problems solved in text. Hand in: p. 122:3, 5, 10; p. 150:11, 21, 23, 25.

MARCH 6

(1) Class discussion on last assignment.

(2) Assignment: Hand in any 10 on p. 150.

MARCH 7

(1) Last assignment discussed.

(2) Assignment: Edgerton and Carpenter, pp. 224–225, handing in 1, 2, 3, 4, 12, 15, 26, 36, 38.
Six weeks' test announced for Wednesday next.

MARCH 10

(1) Class discussion on the following, after the pupils had worked them out in class:

(a) $\frac{x+2}{x-4} + \frac{x+3}{x-2} = 5;$

(b) $\frac{x}{3} - \frac{x^2-5x}{3x-7} = \frac{2}{3};$

(c) $\left(\frac{m+n}{m-n} + \frac{m^2+n^2}{m^2-n^2}\right) \div \left(\frac{m-n}{m+n} - \frac{m^2+n^2}{m^2-n^2}\right)$

(2) Assignment: Edgerton and Carpenter, p. 96:19, 23, 27, 32; p. 100, 14, 16, 23; p. 101:26, 30.

MARCH 11

(1) Class discussion on fractions and on quadratic equations.

(2) Assignment: (a) In how many ways can *you* solve $x^2 - 6x - 16 = 0$?

(b) Solve the general quadratic equation

$$ax^2 + bx + c = 0.$$

(c) If the coefficient of x^2 is 1, to "complete the square," take half the coefficient of x, square it, and add the result to both sides of the equation.

MARCH 12

(1) Six weeks' test.

1. Express $\frac{5}{13}$ as a decimal. Comment.

2. Perform the work indicated:

$$\frac{x}{(x+3)(x-1)}+\frac{x-1}{(x+3)(2-x)}-\frac{x-3}{(2-x)(1-x)}$$

3. Perform the work indicated:

$$\frac{x^4-y^4}{x^2y^2}\left(\frac{x}{x^2-y^2}-1+\frac{y}{x^2+y^2}\right)$$

4. Solve $\frac{2x+7}{2x-7}+\frac{3x-2}{x+2}=5$. (Three methods).

5. Solve $ax^2+bx+c=0$. Pledge.

(2) Assignment: E. & C. Read text carefully, pp. 148–154. Hand in pp. 154–155:1–20.

MARCH 13 AND 14

(1) Class discussion on Exponents. Why are exponents used? $a.a.a.a.a$ can be briefly expressed as a^5. *Hence positive integral exponents.* Operations performed by means of these positive integral exponents are four in number, and are to be explained by use of the above definition.

Operation		*Method*
Multiplication	$a^2.a^3=a^5$	Add exponents
Division	$a^6\div a^2=a^4$	Subtract
Involution	$(a^3)^2=a^6$	Multiply
Evolution	$\sqrt{a^6}=a^3$	Divide

Why do other types of exponents arise? In order for the laws used in the four above operations to be invariable in application.

In division $\dfrac{a^6}{a^4} = a^2$, but what does $\dfrac{a^4}{a^6}$ equal?

$\dfrac{a^4}{a^6}$ can be reduced to $\dfrac{1}{a^2}$, but the *law* for division can be applied here also if meaning is given to negative exponents; i.e., $\dfrac{a^4}{a^6} = a^{4-6} = a^{-2}$; that is, $\dfrac{1}{a^2}$ can be written a^{-2}.

The *negative exponent* then *means* the reciprocal with a positive exponent. Again, $\dfrac{a^6}{a^6}$ can be expressed by the *law* as a^{6-6} or a^0, if the zero exponent be defined. Naturally a^0 is defined to be **1**. Moreover, in applying the law of exponents in performing evolution, a new type of exponent arises. Thus, since $\sqrt[3]{a^6} = a^{6/3} = a^2$, it is natural to say that $\sqrt[3]{a^5} = a^{5/3}$. This new type, the fractional exponent, can be used if it is defined. Obviously, the numerator of the fractional exponent indicates a power while the denominator indicates a root.

(2) Assignment: E. & C., pp. 158–159. Hand in p. 158:12, 13, 19, 21; p. 159:26, 27.

MARCH 17

(1) Quiz: $[(x^{-2/3})^3]^{-(1-1/2)} = ?$
(2) Discussion on negative and fractional exponents.
(3) Assignment: E. & C., p. 153 for oral work. Hand in p. 162:37, 38, 39, 40, 51–67.

MARCH 18

(1) Quiz:
(a) $(-32)^{-2/5}$; (b) $(16)^{5/2}$; (c) $\{(a^m)^{m-1/m}\}^{1/m+1}$

(2) Discussion on quiz and p. 153.

(3) Assignment: Study thoughtfully E. & C., pp. 163–168. Hand in p. 166:34–43; p. 168:28–37.

MARCH 19

(1) Quiz same as last time except for the addition of $-(-32)^{-2/5}$.

(2) Worked out in class p. 155:32, 33, 34, 35.

(3) Assignment: Hand in p. 166:41, 42; p. 169:45–60 Please do not refer to answers for this lesson.

MARCH 20

(1) Class discussion.
Imaginary numbers: $(-16)^{1/4}$; $(16)^{1/4}$; $(-32)^{1/5}$; $(32)^{1/5}$.
"To simplify a radical" means that the radicand must be made (1) integral, and (2) as small as possible.
Consider $a^n.a^{-n}$; $a^2.a^{-2}$; $a^{2/3}.a^{-2/3}$.
Note that $2^2.2^2 = 2^4 = 16$ and $3^2.3^3 = 3^5 = 243$.

(2) Assignment: Speed and accuracy test on pp. 153, 154, 166, 168.

MARCH 21

(1) Quiz: Perform work indicated:

1. $\left(\dfrac{a^{1/2}b^{1/2}}{a^{-2/3}}\right)^0$; 2. $(\sqrt{3a^2b^3})^0.6(ab^2)^0$;

3. $y^{4n+1} \div y^{4n-1}$.

(2) Discussion on exponents and quadratics.

(3) Assignment: E. & C., p. 147:28; p. 255:20, 30 (three ways). Race on p. 154.

MARCH 24

(1) Quiz. Simplify: 1. $[(a^2)^3]^{1/6}$

2. $\dfrac{a^2 - b^{-2}}{a - a^{-1}}$

3. Factor $x^{-2} + 2x^{-1} - 3$.

(2) Assignment: Hand in p. 146:20; p. 225:29.
Simplify:

1. $\sqrt[3]{81b^7c^5/36d}$;
2. $\sqrt[4]{36 \times 32 \times 16}$;
3. $\sqrt[5]{15a^9b^5c^{19}}$;
4. $\sqrt{172b/32a}$;
5. $\sqrt{abc/2xyz}$;
6. $\sqrt[3]{1/27m^2n^3}$;
7. $\sqrt{1250a/4b}$;
8. $\sqrt[3]{5m^2n^5/18d}$;
9. $\sqrt[5]{3b^5c^7/16a}$;
10. $\sqrt[3]{125x/7a}$.

Review pp. 154–155.
In preparation for the 3 weeks' test, how will you review Exponents? Radicals? See previous outline of Exponents. Also p. 165, E. & C.

MARCH 25

(1) Class discussion on radicals, especially the meaning of simplification of radicals.

(2) Assignment: Review pp. 153–156. Study p. 165, illustrating each heading. Hand in: p. 146:24; p. 225:30, and the following:

1. $\sqrt[4]{16ax/25a^2b^2}$
2. $\sqrt[3]{a^9b^7/100}$
3. $\sqrt{99a^5/121b^3}$
4. $\sqrt[3]{576z^9/16}$
5. $\sqrt{ab^{12}/27a^3}$
6. $\sqrt{264a^9b^7/7c}$
7. $\sqrt[3]{90z^4/25a^2b^2}$
8. $\sqrt[5]{e^{14}m^{17}n^{28}/z^4}$
9. $\sqrt[4]{36e^2/169}$
10. $\sqrt[5]{18b^{12}/27a}$

MARCH 26

(1) Quiz:
1. Solve in three ways: $2x^2 - x - 3 = 0$.

2. Simplify: (a) $\dfrac{1}{\sqrt[3]{64}}$; (b) $\dfrac{1}{\sqrt[3]{16}}$.

3. Simplify $(x^3\sqrt[3]{-x})^{-1/2}$.

(2) Assignment: If not thoroughly familiar with p. 154, learn it now. Hand in p. 171:1–20.

MARCH 27

(1) Quiz. Simplify:

1. $\sqrt[3]{8/3}$

2. $\dfrac{\sqrt[3]{8}}{\sqrt[3]{3}}$

3. $\sqrt[3]{9/8}$

4. $\sqrt[3]{24/343}$

5. $2\sqrt[3]{5/3}$

6. $\dfrac{2\sqrt[3]{5}}{\sqrt[3]{3}}$

7. $3\sqrt[5]{2/81}$

8. $2\sqrt[5]{3/16}$

(2) Assignment:
Learn §§80, 81, pp. 173–174.
Hand in pp. 175–176:28–40, 6–16.

MARCH 28

(1) Quiz:

1. $\sqrt{a^4c/b^3}$

2. $\dfrac{\sqrt{a^4c}}{\sqrt{b^3}}$

3. $\sqrt[4]{b^4/a^3}$

4. $\dfrac{\sqrt[4]{b^4}}{\sqrt[4]{a^3}}$

5. $\sqrt[3]{ax^4/b^2}$

6. $\sqrt[3]{7a/125x}$

7. $\dfrac{\sqrt{a^2y^2}}{\sqrt{bd^2}}$

8. $\dfrac{\sqrt[3]{ax^4}}{\sqrt[3]{b^2}}$

(2) Class discussion.

§73. Simplification of Radicals.

1. §74. Extract root: $\sqrt[5]{1/32} = 1/2$.
2. §75. Reduce order:

   $\sqrt[4]{169} = 13^{2/4} = 13^{1/2} = \sqrt{13}$.
3. §76. Make radicand as small as possible: $\sqrt{192} = \sqrt{8^2.3} = 8\sqrt{3}.$
4. §77. Make radicand integral:

   $\sqrt[6]{1/32} = \sqrt[6]{1/32.2/2} = 1/2\sqrt[6]{2}.$
5. §82. Remove radicals from denominator. **Learn rule.**

$$\frac{\sqrt{2}}{\sqrt[3]{5}} = \frac{\sqrt{2}\,.\,\sqrt[3]{5^2}}{\sqrt[3]{5}\,.\,\sqrt[3]{5^2}} = 1/5\sqrt{2}\sqrt[3]{25}$$

Consider also here §84 with its Rule

$$\frac{1}{\sqrt{3}-\sqrt{2}} = \frac{\sqrt{3}+\sqrt{2}}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})} =$$

$$\sqrt{3}+\sqrt{2}$$

§78. Other operations on radicals.

1. §79. Addition of radicals.

   $3\sqrt{2} + 2\sqrt{2} = 5\sqrt{2}$

   Note that only like quantities can be added (or subtracted).

2. §80. Multiplication of radicals.

   $\sqrt{2}\,.\,\sqrt{3} = \sqrt{6}$

   Note that here the *order* of the radicals is the same.

3. §81. Change of order. *Rule.*

   $$\sqrt{5}\,.\,\sqrt[3]{2} = 5^{1/2}\,.\,2^{1/3} = 5^{3/6}\,.\,2^{2/6} = \sqrt[6]{125\,.\,4}$$
   $$= \sqrt[6]{500}$$

4. Division of radicals.

   (See 5 in outline above).

(3) Assignment: Hand in an illustration of every point in outlines above. Study pp. 177–181.

MARCH 31

(1) Quiz:

1. Simplify: $\sqrt{3a^2 + (a/2)^2}$
2. Simplify: $\sqrt[n]{x^{3n}/y^{n-1}}$
3. Simplify: $81^{-1/2} + (-32)^{4/5} + 9^{3/2} + 1{,}564^0$
4. Solve: $1/3^{-x} = 27$

5. Simplify: $\left(\sqrt{4+\sqrt{25}}\right)\left(\sqrt{16-\sqrt{49}}\right)$

(2) Assignment:
Study carefully "Review," p. 177.
Study carefully §§82, 83, 84.
Hand in p. 181:10–14; p. 184:1–4.

Compare $\sqrt{4+9}$ and $\sqrt{x^2+y^2}$;

$\sqrt{4\times 9}$ and $\sqrt{x^2y^2}$;

$\sqrt{x^2+y^2}$ and $\sqrt{x^2+2xy+y^2}$.

APRIL 1

(1) Quiz. Simplify:

1. $\dfrac{2\sqrt{3}}{5}\times 4/9\sqrt{5}\div 6/7\sqrt{2}$ (w)

2. $(x^{18a}\cdot x^{-12})^{1/3a-2}$ (w)

3. $\dfrac{2}{\sqrt{5}-\sqrt{2}}$

(2) Class discussion on quiz and on a comparison of

$\dfrac{x^{1/2}-y^{1/2}}{x^{1/4}+y^{1/4}}$ with $\dfrac{3^{1/2}-2^{1/2}}{3^{1/4}+2^{1/4}}$; and of $4a^{1/2}$ with $(4a)^{1/2}$.

(3) Assignment. Wentworth, p. 367; and quadratics solved by three methods.

APRIL 2

(1) Quiz:

1. Simplify $\dfrac{2\sqrt{10}}{3\sqrt{27}}\times\dfrac{7\sqrt{48}}{5\sqrt{14}}\div\dfrac{4\sqrt{15}}{15\sqrt{21}}$

2. Simplify $[(x^{3ab})^3\times(x^{3b})^{-2}]^{1/(3a-2)}$

3. Simplify $\dfrac{1}{\sqrt{3}+\sqrt{5}}+\dfrac{1}{\sqrt{3}-\sqrt{5}}-\sqrt{3}$

4. Simplify $\dfrac{2\sqrt[4]{128}}{3\sqrt[4]{32}}$

5. Simplify $\dfrac{\sqrt{2}}{\sqrt[3]{2}}$

6. Simplify $\sqrt[4]{2} \,.\, 2\sqrt[4]{32}$

7. Simplify $\sqrt[n]{x^{4n}/y^{3n-2}}$

8. Simplify $\sqrt{a^2 + a^2/4}$

9. Solve $2^{-x} = 32$

10. Simplify $\dfrac{x^{-3} - y}{x^{-1} - y^{1/3}}$

11. $[a^{1/2}x^{-1/2}/(ax)^{2/3} \div a^{-1/2}x/x^{2/3}a^0]^{-1}$

12. Solve in three ways: $x^{-2} - 2x^{-1} - 3 = 0$.

(2) Assignment: "Race" on exercises in §86. Hand in pp. 185–186, "Review"; p. 189:7, 12, 14, 17, 1(e), 4(d); p. 190:5(d), 6(f).

APRIL 3

(1) Class discussion on last quiz.
(2) Assignment: Rework last assignment. Hand in examination questions solved. Study §86. Work problems, p. 445.

APRIL 4

(1) Class discussion on last assignment.
(2) Assignment: Hand in pp. 444:18, 19, 23, 24, 25; pp. 445:27; p. 433:233, 235, 227; p. 432:221; p. 430:206; p. 428:189; p. 426:169; p. 423:140. In working this assignment check each problem carefully with the outlines previously given on Exponents and Radicals.

APRIL 7

(1) Quiz. Simplify $(x^{p+q}/x^q)^p \div (x^q/x^{q-p})^{p-q}$
(2) Class discussion on last assignment.

(3) Assignment: Short quiz on p. 156.
Hand in Review, pp. 218–219; p. 423:140; p. 426: 169; p. 428:189.

APRIL 8

(1) Quiz.

1. Simplify $$\frac{1}{\sqrt{5}-\sqrt{7}}+\frac{1}{\sqrt{5}+\sqrt{7}}+2\sqrt{5}$$

2. Simplify $\sqrt{x^2/9+x^2}$

(2) Discussion of quiz and of last assignment.

(3) Assignment: Quadratics, pp. 220–225.
Hand in pp. 222:22–28; p. 225:31–35.

APRIL 9

(1) Quiz. Simplify $$\frac{a-b}{a^{1/3}-b^{1/3}}-\frac{a+b}{a^{1/3}+b^{1/3}}.$$

(2) Class discussion on exponents and quadratics.

(3) Assignment: §§101, 102. Note discussion.
Hand in p. 230:6–19.
Quiz on p. 155, first column.

APRIL 10

(1) Quiz.

1. Simplify $\sqrt{x^{-2}z^{-1}} \cdot \sqrt[3]{x^{-1}\sqrt{z^3}}$
2. Simplify $(a^{n+1}/a^{1-n})^n \div a^{-n}/a$

(2) Discussion on quiz and p. 230.

(3) Assignment: Study pp. 227–230.
Hand in p. 230:6–26.
Quiz on p. 189 and p. 155.

APRIL 11

(1) Class discussion on radical equations.

Consider (1) $\sqrt{x+9}-\sqrt{x}=9$

(2) $-\sqrt{x+9}-\sqrt{x}=9$

(3) $-\sqrt{x+9}+\sqrt{x}=9$

(4) $\sqrt{x+9} + \sqrt{x} = 9$

In each of these equations, the value found for x is 16. Does 16 satisfy these equations? Why is 16 found in (1), (2), and (3)? Can any one of these four equations be solved without solving the other three?

(2) Assignment: Wentworth, p. 280.
Hand in any ten examples. Quiz on the 14th on p. 189 or radical equations. Note examples solved on pp. 227, 228, 229.

APRIL 14

(1) Quiz: $4x + 1 - 3\sqrt{4x+5} = 6$.
(2) Class discussion on radical equations.
(3) Assignment: Study pp. 232–234.
Hand in Review, p. 231; p. 234:1–4.
Quiz on p. 229:7–16; p. 117:36–44; p. 182:21–25.

APRIL 15

(1) Quiz. Simplify $\sqrt[2n]{64^n/4^{2n}}$.
(2) Class discussion on last assignment.
(3) Assignment: E. & C. Review pp. 229–230.
Hand in p. 234:5–18.

APRIL 16

(1) Quiz:

1. Solve $\sqrt{x} - \sqrt{x-12} = 2$
2. Solve $\sqrt{9x^2+5} - 3x = 1$
3. Solve $\sqrt{x+5} + \sqrt{x-3} = 2\sqrt{x}$.

(2) Class discussion on quiz.
(3) Assignment: Hand in E. & C., p. 235:19–30.

APRIL 17

(1) Quiz. Simplify $[\sqrt{72y^{2n}}/3 \times 9^0]\,(2y^{n+2})^{-1}$.
(2) Class discussion on assignment.
(3) Assignment: p. 235:31–34; "Review."
For quiz: Chapters VI and VII.

APRIL 18

(1) Quiz. Simplify $\dfrac{a^{1/2}b^{1/4}}{a^{1/2}+b^{1/2}} \cdot \dfrac{a-b}{a^{1/4}+b^{1/4}}.$

(2) Class discussion: Wentworth, p. 235.

(3) Assignment: Wentworth, p. 283:7, 8, 10, 13, 16–22.

APRIL 22

(1) Quiz. Simplify $(2^{1/n-1} \,.\, 2^{1/n+1})^{1/n}$

(2) Class discussion on W., p. 283.

(3) Assignment: Hand in any 8 from p. 230:18, 20; p. 231:1, 3, 6; pp. 234–235:10, 11, 15, 18, 20, 28, 34.
Six weeks' quiz on exponents, radicals, quadratic form.
Good reviews on p. 219, p. 189. Note p. 452:90.

APRIL 23

(1) Six weeks' test:

1. Simplify $\dfrac{x^{3/2}-1}{x^{1/2}-1} - \dfrac{x-1}{x^{1/2}+1}$

2. Solve $\sqrt{2x-3} - \sqrt{x+1} = \sqrt{5x-4}$

3. Solve $x^{2/3} - x^{-2/3} = 17/4.$

These three form the quiz. If any one has time, credit will be given for working the following:

4. Solve $2x^2 + 3x - 5\sqrt{2x^2+3x+9} = -3$
5. Simplify $(2^{n+4} - 2 \times 2^n) \times (2^{-2} \times 2^{-n-3})$
6. Solve $8x^{-3} + 19x^{-3/2} - 27 = 0.$

(2) Assignment: Hand in pp. 411–413:2, 3, 6, 11, 16, 19, 30; p. 238:1, 2.

APRIL 24

(1) Quiz. Find value of $(125)^{-2/3} + \dfrac{3}{2+2^{-1}}$

(2) Class discussion on last assignment.

(3) Assignment: p. 238:1–10.

APRIL 25

(1) Quiz. Simplify $\dfrac{3^n(3^{n-1})^n}{3^{n+1}3^{n-1}9^{-n}}$

(2) Class discussion on exponents and p. 238.

(3) Assignment: Hand in any ten on pp. 238–240. Work five more on these pages.

APRIL 28

(1) Class discussion on pp. 238–240.

(2) Assignment: Quiz on E. & C., p. 155.
Hand in Wentworth, p. 154:2, 3; 155:7, 8; 156:14, 15; 157:24, 25; 158:30, 31; 159:37, 38; 160:43, 44.

APRIL 29

(1) Quiz. Simplify $\dfrac{a^{-2}x^{-2}}{(ax)^{-1}-(ax)^2}$

(2) Class discussion on word problems.

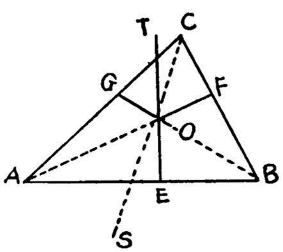
(3) Assignment: Quiz on E. & C., p. 156.
Hand in Wentworth, p. 161:48–51; p. 162:55, 56; p. 163:62–68.

APRIL 30

(1) Prove

$$\begin{aligned} 4 &= 1 \\ 8 &= 8 \\ 2 &= 2 \\ 8-2 &= 8-2 \\ 8-8 &= 2-2\text{; if we divide by } 2-2. \\ 4 &= 1.\ \text{Where is the flaw?} \end{aligned}$$

Prove that all $\triangle$ are isosceles.



Make $\angle ACS = \angle BCS$
Make $AE = EB$
Draw $ET \perp$ to AB
CS and TE meet at O
Draw OF and OG $\perp$ to CB and AC, respectively,
$\triangle CGO = \triangle COF$ (Why?)
Then $\triangle AOG = \triangle BOE$ (Why?)

CG = CF (Why?)
AG = BF
AC = CB. Where is the flaw?
Also discussion on word problems.

(2) Assignment: Hand in E. & C., p. 128:1; p. 242, Review 1, 2, 3, 4, 5.
Test on exponents. Study pp. 155–156.

MAY 1

(1) Quiz:

1. Simplify $\dfrac{a^n \cdot (a^{n-1})^n}{a^{n+1}a^{n-1}}$

2. Simplify $\dfrac{10^n \times (10^{n-1})^n}{10^{n+1} \times 10^{n-1}}$

or $\dfrac{2^n \times (2^{n-1})^n}{2^{n+1} \times 2^{n-1}}$

(2) Class discussion on logarithms.

(3) Assignment: E. & C., pp. 185–186:2–5; p. 281:2; p. 129:44, 47.

MAY 2

(1) Quiz. Solve either
The denominator of a fraction is 6 more than the numerator. What is the fraction if its value is .25?

or,

The hypotenuse of a right triangle is 20. The sum of the other two sides is 28. Find the lengths of the sides.

(2) Class discussion on logarithms.

(3) Assignment: Hand in p. 281:3, 4, 5; p. 189, Review.

MAY 5

(1) Quiz. Solve either *1* or *2*.

1. The length of a rectangle exceeds its width by 8 ft. If each dimension were 2 ft. more, the area would 125 sq. ft. more. Find its dimensions.

2. Find the time between 1 and 2 o'clock when the hands of a clock point in opposite directions.

(2) Class discussion on logarithms.

(3) Assignment: E. & C., pp. 194–195.
Study for quiz Wentworth's fractional equations.

The following is the discussion on logarithms that was given in class on May 1, 2, 3, and 5.

A *logarithm* is an *exponent.*
$10^2 \cdot 10^0 \cdot 10^3 = 10^5$
$10^2 = 100$; 2 is the log. of 100 to the base 10.
$10^3 = 1000$; 3 is the log. of 1000 to the base 10.
$10^0 = 1$; 0 is the log. of 1 to the base 10.

$10^{-2} = \dfrac{1}{100}$; -2 is the log. of .01 to the base 10.

$10^{-4} = \dfrac{1}{10000}$; -4 is the log. of .0001 to the base 10.

Or, the log. of 1000 is 3, etc.

What do you know about the logarithms of numbers between 10 and 100?

E. g., $10 < 25 < 100$
Or $10^1 < 10^x < 10^2$
Hence $1 < x < 2$, or, $25 = 10^{1+}$

The table records the decimal part of the log. Hence from the table $25 = 10^{1.3979}$.

Consider 625:
$100 < 625 < 1000$
$10^2 < 10^x < 10^3$; $2 < x < 3$, $\therefore$ $625 = 10^{2+}$.
From the tables then $625 = 10^{2.7959}$.
But $(25)^2 = 625$, $(10^{1.3979})^2 = 10^{2.7958}$ (note difference in last digit).
$100 < 243 < 1000$
$10^2 < 10^x < 10^3$; $2 < x < 3$, $\therefore$ $243 = 10^{2+}$,
and from the table we get the decimal, or $243 = 10^{2.3856}$.
$625 \times 243 = 10^{2.7959} \times 10^{2.3856}$
$\therefore$ $151875 = 10^{5.1815}$.

Verify by finding log. of 151875 approximately.
$151000 = 10^{5.1790}$
$152000 = 10^{5.1818}$.
Therefore, presumably, the above result is correct.

Get log. 54.2, log. 542; log. 57.6, log. 576.
Check by the products 54.2×10, 57.6×10.

Interpolation:

$$100\left[\begin{array}{l} 27\left[\begin{array}{l} 86500 \ldots\ldots 9370 \\ 86527 \ldots\ldots (?) \end{array}\right] q \\ \quad\; 86600 \ldots\ldots 9375 \end{array}\right] 5$$

By principle of proportional parts:

$$\frac{q}{5} = \frac{27}{100}, \quad q = \frac{27 \times 5}{100} = 1.35;$$

since $10000 < 86527 < 100000$,

$$86527 = 10^{4.9371}.$$

Assignment: Get logarithms of

1. $\dfrac{683 \times 65245}{6794}$

2. $\dfrac{3768 \times 672}{538}$

$46 = 10^{1.6628}$	Get log 828:
$18 = 10^{1.2553}$	$100 < 828 < 1000$
	$828 = 10^{2+}$
$828 = 10^{2.9181}$	$828 = 10^{2.9180}$

Note difference in last digit.

To find the number corresponding to a given logarithm.

$$\frac{86257}{325} = \frac{10^{4.9371}}{10^{2.5119}} = 10^{2.4252}$$

$$16\left[\begin{array}{l} 3\left[\begin{array}{l} 4249 \ldots\ldots 266 \\ 4252 \ldots\ldots (?) \end{array}\right] q \\ \quad\; 4265 \ldots\ldots 267 \end{array}\right] 1$$

Then $3:16 = q:1$, and $q = \dfrac{3}{16} \times 1 = .2$,

$\therefore$ 266.2 is the number required.

Since $10^{2.4252}$ is greater than 100 and less than 1000, $10^{2.4252} = 266.2$.

$$\text{Better, } 16 \quad 3 \begin{bmatrix} 4249 \dotfill 2660 \\ 4252 \dotfill (?) \end{bmatrix} q \qquad 10$$
$$4265 \dotfill 2670$$

$\dfrac{3}{16} = \dfrac{q}{10}$, and $q = 2$.

Hence 2662 is the succession of digits corresponding to 4252 as the decimal part of the logarithm.

The required number is $10^{2.4252}$ and hence $100 < N < 1000$, $\therefore 10^{2.4252} = 266.2$.

MAY 6

(1) Quiz. Simplify:

1. $$\frac{\dfrac{1}{1-a} + \dfrac{1}{1+a}}{\dfrac{a}{1-a} + \dfrac{1}{1+a}}$$

2. $2/5\sqrt{3} \times 4/9\sqrt{5} \div 6/7\sqrt{2}$

3. Find by logarithms $\dfrac{263 \times 9643}{567}$.

(2) Discussion of logarithms.

(3) Assignment: "Reviews" on pp. 177–178, 183, 185, 189–190, 195, E. & C. Hand in any of these that you have previously failed to hand in.

MAY 7

(1) Class discussion.

$$\frac{263 \times 9643}{567} = \frac{10^{2.4200} \times 10^{3.9842}}{10^{2.7536}} = 10^{3.6506} = 4473.$$

To explain 4473:

$10^3 < 10^{3.6506} < 10^4$

$$10\left(3\left(\begin{array}{l}6503 \ldots\ldots 44700\\ 6506 \ldots\ldots 447300\end{array}\right)q \quad \begin{array}{l}\\ \\ 6513 \ldots\ldots 448000\end{array}\right)1000$$

$$\frac{3}{10} \times 1000 = 300.$$

This illustrates the fact that any number of 0's can follow 447 and 448. Many such examples were worked in class by the students.

Consider also the following:

$$\frac{653 \times 42}{86738} = \frac{10^{4.4381}}{10^{4.9382}} = 10^{-.5001}.$$

Since the tables give no negative decimals, $-.5001$ can be written in the equivalent form $-1 +$ .4999, or $\bar{1}.4999$. We can subtract thus:

$$\begin{array}{r} -1 + 5.4381\\ 4.9382\\ \hline \bar{1}.4999\end{array}$$

To find the corresponding number:

$$14\left(2\left(\begin{array}{l}4997 \ldots\ldots 31600\\ 4999 \ldots\ldots 31614\end{array}\right)q \quad \begin{array}{l}\\ \\ 5011 \ldots\ldots 31700\end{array}\right)100$$

$$q = \frac{2}{14} \times 100 = \frac{100}{7} = 14 +$$

$$\text{Now } \frac{316.14}{10^3} = \frac{10^{2.4999}}{10^3},$$

and $.31614 = 10^{\bar{1}.4999}$.

(2) Assignment: Long division with expressions having exponents. Also E. & C., p. 252:1, 2; p. 235:3, 4, 5, 6, 7, 9, 10; p. 281, 7.

MAY 8

(1) Quiz.

1. Divide $1/x^{3/5} - 1/y^{3/5}$ by $\sqrt[5]{x} - \sqrt[5]{y}$.
2. What is the log. of $(2134)^{2/3}$?

(2) Class discussion.

Find $\dfrac{364}{895}$ by logarithms.

$$\frac{364}{895} = \frac{10^{2.5611}}{10^{2.9518}} = 10^{\bar{1}.4999}.$$

To perform the subtraction, we write

$$\begin{array}{r} -1 + 3.5611 \\ 2.9518 \\ \hline \bar{1}.6093 \end{array}$$

We find $10^{1.6093} = 40.673$, but we want $10^{\bar{1}.6093}$; hence we divide this equation by 10^2 and have

$$\frac{10^{1.6093}}{10^2} = \frac{40.693}{10^2}$$

$$10^{\bar{1}.6093} = .40693$$

(3) Assignment: Look over "Reviews" on pp. 208–210, E. & C. Hand in "Review," pp. 211–212:2, 3, 4, 5, 6, 7, 8, 9, 11, 15.

MAY 9

(1) Class discussion:

The description of an equation in quadratic form is important. Note—

1. $8x^6 + 64x^3 = 8$, a quadratic in x^3.
2. $\sqrt{x^3} - 3\sqrt[4]{x^3} = 40$, a quadratic in $x^{3/4}$.
3. $(2x - 3)^2 - (2x - 3) = 6$, a quadratic in $2x - 3$.

4. $7x^2 - 5x + 8\sqrt{7x^2 - 5x + 1} = -8$ can be written as a quadratic in $\sqrt{7x^2 - 6x + 1}$: $(7x^2 - 5x + 1) + 8\sqrt{7x^2 - 5x + 1} + 7 = 0$.
5. $x^{1/2} - 13/6 + x^{-1/2} = 0$ can be written $6x - 13x^{1/2} + 6 = 0$, a quadratic in $x^{1/2}$.
6. $3\sqrt[4]{x} - 2\sqrt{x} = -20$ or $2x^{1/2} - 3x^{1/4} - 20 = 0$, a quadratic in $x^{1/4}$.
7. $6x^2 + 6x + \sqrt{x(x+1)} = 7$, a quadratic in $\sqrt{x(x+1)}$.

(2) Assignment: Rework, laying book aside, problem worked out on p. 281. Hand in p. 376:22, 25, 26, 27, 32; p. 281, 9; p. 287, 21; p. 447, 55; p. 423:146.

MAY 12

(1) Class discussion.
Multiply by logs:
$16.73 \times 2.854 = 10^{1.2235} \times 10^{0.4554} = 10^{1.6789} = 47{,}744$.

$$9\left[\begin{array}{l} 4\left[\begin{array}{l} 6785 \ldots\ldots 47700 \\ 6789 \ldots\ldots 47744 \end{array}\right] q \\ \quad 6794 \ldots\ldots 47800 \end{array}\right] 100$$

$$\frac{q}{100} = \frac{4}{9}, \quad q = \frac{400}{9} = 44.4$$

(2) Assignment. Review as many recent assignments as you can for the quiz.

MAY 13

Club day.

MAY 14

(1) Three weeks' quiz:
1. Solve $4x^{-2/3} - 3x^{-1/3} - 27 = 0$.
2. Solve $\sqrt{x+4} - \sqrt{x} = \sqrt{x + 3/2}$.

3. Solve $\begin{cases} x - y = 4, \\ x^2 + y^2 = 40 \end{cases}$ or $\begin{cases} x^3 + y^3 = 18xy, \\ x + y = 12 \end{cases}$

4. Solve: A and B can do a piece of work together in 18 days and it takes B 15 days longer to do it than it does A. In how many days can each do it alone? or
The area of a square may be doubled by increasing its length by 10 ft. and its breadth by 3 ft. Find the length of its side.

5. A substitute for any one of the above 4:

$$(x^{n/m-n})^{m^2-n^2} \div (x^n/x^m)^n$$

(2) Assignment to be handed in: E. & C., p. 241:28, 29; p. 283:11, and Review. Note carefully p. 282, §121.

MAY 15

(1) Quiz. Solve $\begin{cases} x^2 - xy + y^2 = 13, \\ x^2 + 3xy + y^2 = 61 \end{cases}$

(2) Class discussion on graphic solutions; graphed in class $x - y = 4$. Also consider the quadratic expression $x^2 - 6x + 5$, and the quadratic equation $x^2 - 6x + 5 = 0$. Let $y = x^2 - 6x + 5$, and note that this graph cuts the x-axis at 1 and 5, the roots of the quadratic equation. Roots real and unequal: $\sqrt{36 - 20} = \sqrt{16} = 4$.

(3) Assignment in Wentworth: p. 367:23, 24, omitting decimals.
Solve by algebra and by graph

$$\begin{cases} x + y = 7, \\ x - y = 1 \end{cases} \text{ and } \begin{cases} xy = 12, \\ x^2 + y^2 = 25 \end{cases}$$

MAY 16

(1) Quiz. Simplify:
1. $(2/x^{n-1} \,.\, 1/x^{n+1})^{(n-1)/n}$;
2. $[(x^m)^{m-1/m}]^{1/(m+1)}$
3. $(x^{m+n}/x^n)^m (n/x^{m+n})^{m-n}$;

4. $(a^{x/(x+y)} \div a^{(x-y)/x})^{(x+y)/y}$;
5. $(a^x/a^{x+y} \div a^{x-y}/a^x)^{(x+y)/y}$.

(2) Class discussion on quiz and on the following:
1. Get the logarithm of .00234.

$$\frac{234}{10^5} = \frac{10^{2.3692}}{10^5} = 10^{\overline{3}.3692}$$

2. Get the logarithm of .0673.

$$\frac{673}{10^4} = \frac{10^{2.8280}}{10^4} = 10^{\overline{2}.8280}$$

(3) Assignment: Construct four quadratics to solve in four ways each, Wentworth, p. 276:28 and 29.

MAY 19

(1) Quiz on high-school algebra.
1. Find the value of $3/5 + 5/2 - 4/7$.
2. Divide 0.02683 by 0.007213 to two decimal places.
3. Simplify:

$$\text{(a)}\ \frac{2+\sqrt{8}}{\sqrt{2}}; \quad \text{(b)}\ \frac{x - 1/x}{x+1}.$$

4. Factor completely:
   (a) $y^4 - 81$; (b) $a^2b + 3a^2 + ab + 3a$
5. Solve: $7x/2 - 11 = 3x/4$
6. Solve simultaneously: $\begin{cases} 2m - 7n = 2, \\ 6m + 4n = 31 \end{cases}$
7. Solve: $3x^2 - x - 14 = 0$.
8. If a and b are the sides and c the hypotenuse of a right triangle, find the side a and the area if $b = 12$, $c = 15$.

(2) Graphed $y = x^2 - 4x - 5$.

(3) Assignment:
1. Kinds of exponents—
   a. Positive integral—why?

$$\left.\begin{array}{l}\text{b. Negative}\\ \text{c. Fractional}\\ \text{d. Zero}\end{array}\right\}\text{—why?}$$

2. Operations—

$$\left.\begin{array}{l}\text{a. Multiplication}\\ \text{b. Division}\\ \text{c. Involution}\\ \text{d. Evolution}\end{array}\right\}\text{ How performed?}$$

Learn outline. Review thoroughly pp. 147–162. Note exponents, pp. 444–445. Hand in four difficult examples.
Plot $y = x^2 - 6x - 7$.
Get log. of .02345.

MAY 20

(1) Quiz. Simplify:
1. $[(x^6)^{3a}(x^{-4})^3]^{1/(3a-2)}$
2. $[\{(a^{-m})^{-n}\}^p]^q \div [\{(a^m)^n\}^{-p}]^{-q}$
3. $(x^{p+q}/x^q)^p \div (x^q/x^{q-p})^{p-q}$

(2) Class discussion on quiz, and graphed $x^2 + y^2 = 25$, or $y = \pm\sqrt{25 - x^2}$.

(3) Assignment:

Graph
$$\begin{aligned} y &= x^2 - 3x + 2\\ y &= x^2 + 4x + 3\\ y &= x^2 + 4x + 4\\ y &= x^2 + 4x + 5 \end{aligned}$$
Solve the quadratics and compare the roots with the graphs.

Perform by logs.: $\dfrac{(.0234)^{2/3}}{.008654}$.

Graph each equation used in solving $\begin{cases} x^2 + y^2 = 25,\\ xy = 12 \end{cases}$

MAY 21

(1) Quiz.

1. Divide $\sqrt[3]{x^2} + \sqrt[3]{x} - 12$ by $\sqrt[3]{x} - 3$
2. Simplify $(16a^{-4}/81b^3)^{-3/4}$
3. Simplify $(3^{b/(a-b)})^{a^2 - b^2} \div (3^b/3^a)^b$
4. What is the value of
$81^{-1/2} + (-32)^{4/5} + 9^{3/2} + 1564^0$?

(2) Class discussion.

Consider $y = x^2 - 3x + 2$

Note that $b^2 - 4ac$ for $x^2 - 3x + 2 = 0$ is $9 - 8 = 1 > 0$

Roots real and unequal (graph drawn).

In $y = x^2 + 4x + 4$, let $x^2 + 4x + 4 = 0$;
$b^2 - 4ac = 16 - 16 = 0$,
and the roots are real and equal (graph drawn).

Then $y = x^2 + 4x + 5$ gives the quadratic equation
$x^2 + 4x + 5 = 0$

whose roots are $\dfrac{-4 \pm \sqrt{16 - 20}}{2} =$

$-2 \pm \sqrt{-1}$; and the roots are "imaginary" (graph drawn).

(3) Assignment to be handed in: Any eight of p. 413: 19, 23, 24, 29, 30; p. 414:34, 39, 41, 42, 43.

Also:

1. $(.00678)^{-1/2} \times (.0345)^{1/3}$
2. Graph equations used in solving

$$\begin{cases} x^2 + y^2 = 169, \\ xy = 60 \end{cases}$$

MAY 22

(1) Class discussion.

$(4a^{-2/3})^{-3/2}$, $(3^{3/8}a^{-3})^{-2/3}$, $(16a^{-4}/81b^3)^{-3/4}$

Also $(.0234)^{2/3} = (10^{2.3692})^{2/3} = 10^{2.9131}$

The actual steps in performing this calculation are:

$$\frac{234}{10^4} = \frac{10^{2.3692}}{10^4}, \text{ or } \frac{23.4}{10^3} = \frac{10^{1.3692}}{10^3}$$

and $.0235 = 10^{\bar{2}.3692}$ and $.0234 = 10^{\bar{2}.3692}$

$$(10^{\bar{2}.3692})^{2/3} = 10^{\frac{(\bar{2}.3692)\times 2}{3}}$$

$$\begin{array}{r} -2 + .3692 \\ 2 \\ \hline 3\ \underline{|-4 + .7384} \end{array} \quad \text{or} \quad \begin{array}{r} 3\ \underline{|-6 + 2.7394} \\ -2 + .9131 \end{array}$$

$\therefore (10^{\bar{2}.3692})^{2/3} = 10^{\bar{2}.9131}$.

(2) Assignment: E. & C., pp. 154–155:15, 16, 19, 22; p. 437:268; p. 432:221; p. 416:65; p. 377:33, 37; pp. 243, 244.

Find the value of $\dfrac{(.00671)^{-2/3}}{(.0234)^{-1/4}}$ by logs.

Solve $\begin{cases} x^2 + y^2 = 169, \\ \quad\; xy = \;\, 60, \end{cases}$ and draw the graphs.

MAY 23

(1) Class discussion on the last assignment.

(2) Assignment: E. & C., Ch. XII; p. 293:3, 19, 24.
Hand in Review, p. 293.
Hand in Review, p. 301, 17.
Review Wentworth, p. 77.
Solve in four ways: $x^2 - 4x - 5 = 0$.

MAY 26

(1) Class discussion on graphs and logarithms.

$$\left(\frac{.548 \times 1.98}{39.6 \times 2.74}\right)^{1/2} = \frac{10^{\bar{1}.7388} \times 10^{0.2967}}{10^{1.5977} \times 10^{0.4378}}$$

$$= \frac{10^{.0355}}{10^{2.0355}} = (10^{-2})^{1/2} = .1$$

(2) Assignment. Hand in the five hardest review problems that you can find, Wentworth, p. 404.

MAY 27

(1) Class discussion.
Worked in class the "hard" problems called for in previous assignment.

(2) Assignment: p. 424, 153; p. 281, 6; p. 240, 19; p. 189, 16; p. 169, 49, 63; p. 171, 12; p. 172, 20; p. 104, 17; p. 185, 17; p. 239, 13.

Solve $\sqrt{x+2} + \sqrt{x-1} - \sqrt{3x+3} = 0$

$$x^{-3/2} - 4x^{-3/4} - 32 = 0$$

$$3x^2 + 15x - 2\sqrt{x^2 + 5x + 1} = 2.$$

MAY 28

(1) Class discussion a general review of the term's work.

(2) Assignment: The final examination given the seniors:

1. (a) Simplify $3\sqrt{5/32} + 2/3\sqrt{40/9} - \sqrt{1/10}$
   (b) Solve and check:

$$\sqrt{5+x} + \sqrt{5-x} = \frac{12}{\sqrt{5-x}}$$

2. Plot and check by algebraic solution:

$$\begin{cases} x^2 + y^2 = 25, \\ x - y + 1 = 0 \end{cases}$$

3. Solve by factoring (a) $8x^2 - 10x = 3$

   (b) $\sqrt[3]{x^2} + 3\sqrt[3]{x} = 1\frac{3}{4}$

4. Solve by two methods: $x^2 - cx = 1/2(ax - ac)$

5. (a) Simplify $\dfrac{2\sqrt{3} - 5}{\sqrt{3} + 5}$

   (b) Simplify $\dfrac{x - 1}{x^{1/3} - 1} - \dfrac{x^{2/3} - 1}{x^{1/3} + 1}$

6. (a) Simplify $[(x^{5ab})^3 \times (x^{5b})^{-2}]^{1/(3a-2)}$

   (b) Simplify $\dfrac{\dfrac{a}{a-b} - \dfrac{a}{a+b}}{\dfrac{b}{a-b} + \dfrac{a}{a-b}}$

7. (a) Simplify $\dfrac{3^{-2} - 2^{-2}}{3^{-1} - 2^{-1}}$

   (b) Solve for l in $t = \sqrt{l/g}$

8. The width of a room is $\frac{3}{4}$ of its length. If the width were 4 ft. more and the length 4 feet less, the room would be square. Find its dimensions.

9. A can do a piece of work in 6 days, B in 5 days, and C in 4 days. How long will it take them all working together to do the work?

10. Find the logarithm of

$$\left(\frac{.0137}{42.85}\right)^{1/3}$$ if 13 in the 7 column is 1367
42 in the 8 column is 6314
42 in the 9 column is 6325

Answer any eight.

Final examination for 11A's.

1. Find by logarithms:
$$\frac{32.48 \times (.288)^3}{\sqrt[4]{525}}$$

2. (a) Simplify $\dfrac{2\sqrt{10}}{3\sqrt{27}} \times \dfrac{7\sqrt{48}}{5\sqrt{14}} \div \dfrac{4\sqrt{15}}{15\sqrt{21}}$

(b) Solve by factoring:
$$4x^{2/3} - 3x^{1/3} - 27 = 0$$

3. Plot and solve algebraically:
$$\left.\begin{aligned} x^2 + y^2 &= 25, \\ xy &= 12 \end{aligned}\right\}$$

Do the solutions check?

4. Simplify (a) $\dfrac{\sqrt{2} - 3}{2\sqrt{5} - 3}$

(b) $\dfrac{a^{2/3} - b^{2/3}}{a^{1/3} + b^{1/3}} - \dfrac{a - b}{a^{1/3} - b^{1/3}}$

5. (a) Simplify $\dfrac{2^{-5}3^4\sqrt{\sqrt[3]{64}}}{(27)^{4/3}\sqrt{1/4}}$

(b) Solve and check:

$$\sqrt{x+15} - \sqrt{x+3} = 2\sqrt{x}$$

6. A can do a piece of work in 8 days, B in 10 days. A and B together with the help of C can do the work in 3 days. How long will it take C alone to do the work?
7. A rectangular grass plot 12 yds. long and 9 yds. wide has a path around it. The area of the path is 2/3 of the area of the plot. Find the width of the path.
8. (a) Simplify $\dfrac{\dfrac{1}{a-b} - \dfrac{b}{a^2-b^2}}{\dfrac{a}{ab+b^2} - \dfrac{b}{ab+a^2}}$

   (b) Solve $7x^2 - 5x + 8\sqrt{7x^2 - 5x + 1} = -8$
9. (a) Simplify $\dfrac{a^2 + b^{-2}}{a^{-2} + b^2}$

   (b) Solve in three ways:
   $2x^2 - 3x - 5 = 0$
   Can you do this a fourth way?
10. (a) Solve $\sqrt{x-6} + \sqrt{x} = 3/\sqrt{x-6}$

    (b) Simplify

    $$3\sqrt{200} - 6\sqrt{1/50} + 3\sqrt{1/15} + 5\sqrt{60}$$
11. Solve (a) $\dfrac{x+1}{x-1} = \dfrac{m+n}{m-n}$

    (b) $\sqrt{x} - \sqrt[4]{x} = 2$

Pledge.

THE THREE FAMOUS PROBLEMS OF GREEK GEOMETRY*

DELLA HOUSSELS
Austin High School

CHAPTER I

HISTORY

The earliest history of mathematics shows that the Babylonians and Egyptians each possessed a highly-developed system of mathematics as early as 2000 B.C. It is not known whether the two systems sprang from a common source or were developed independently. It is known that about 2200 B.C. the Babylonians had a wide knowledge of the subject and, in some respects, had developed it to a higher degree than had the contemporary Egyptians.

Babylonian tablets are now in the British Museum, the Prussian State Museum at Berlin, the Ottoman Museum at Constantinople, the University of Strasbourg, the University of Pennsylvania, and the Palais de Cinquantenaier of Brussels. One of these tablets, of about 2200 B.C. reveals the fact that the Babylonians, even at that early date, knew how to find the area of a rectangle, of a right triangle, and of a trapezoid. An Akkadian tablet (about 2000 B.C.) gives a method for finding the diagonal of a rectangle which suggests that the Babylonians may have known the Pythagorean theorem long before Pythagoras was born. Various problems on these tablets show that 3 was then taken as the value of π. The same value of π was used at a later date (560 B.C.) by the Hebrews (I Kings, 7:23, II Chronicles, 4:2).

The earliest known record of Egyptian mathematics is a papyrus called the Golenishchev Papyrus. It was written about 1850 B.C. and is now in the Museum of Fine Arts at

---

*We are publishing Mrs. Houssels' thesis because of its general interest. The remainder of the thesis will be given in subsequent issues of the Bulletin.

Moscow. It contains formulae for finding the volume of a hemisphere and the volume of a frustrum of a regular square pyramid.

A second papyrus, called the Rhind Papyrus, is in the British Museum. It was written about 1650 B.C. by the Egyptian priest Ahmes, and bears the interesting title "Directions for Knowing All Dark Things." It contains problems for finding the area of a circle and the relation between the area of a circle and that of the circumscribed square; finding the area of the part of a triangle cut off by a line parallel to one of the sides; problems dealing with the volumes of cylinders and parallelepipeds; and a discussion of the angles formed between the base and the faces of a pyramid. Both of these papyri are thought to be copies of earlier documents of about 2000 B.C.

Geometry was first introduced into Greece by Thales of Miletus (640–550 B.C.), one of the "Seven Wise Men of Greece." He studied for many years in Egypt and later founded the Ionic school of philosophy in Greece. He added many important theorems to those already known to the Egyptians. His pupil Pythagoras (569–500 B.C.) also studied in Egypt and founded a school at Croton, Italy, which advanced geometry to the status of a liberal science.

Euclid (about 330–275 B.C.), a Greek geometer, taught at Alexandria. Little is known of his life except that he wrote a geometry textbook called "The Elements," which is one of the most famous books ever written. It is a compilation of the works of earlier writers, assembled and arranged in so logical an order that little change was made in it for hundreds of years. This book has formed the basis for all later textbooks on the subject.

Archimedes (287–212 B.C.), the greatest mathematician the world has known, except Newton, lived at Syracuse in Sicily. The following books written by him survive: *Equiponderance of Planes; Quadrature of the Parabola; The Sphere and Cylinder; Measurement of the Circle; Spirals; Conoids and Spheroids; The Sand-Counter;* and *Floating Bodies.* He used both circumscribed and inscribed regular polygons and proved that the circle is the limit which each perimeter approaches as the number of sides is indefinitely increased. He proved that the area of a circle equals πr^2; that $\pi r^2 : (2r)^2 :: 11 : 14$ very nearly; that $3\ 1/7 > \pi > 3\ 10/71$; and reduced the problem of the quadrature of the circle to the construction of π.

The later Greek geometers were interested in what were known as "The Three Famous Problems."

1. The duplication of the cube, i.e., finding the edge of a cube which shall have twice the volume of a given cube.
2. The trisection of any angle.
3. The quadrature of the circle, i.e., the construction of a straight line segment equal in length to a given circle or, the construction of a rectilineal figure equal in area to a given circle.

The solution of these problems with the instruments then in use—the straight edge and the compass—has baffled all who have attempted them. However, persistent efforts to find solutions to them have led to the discovery of many new theorems and have developed geometry into a wonderful and beautiful science.

The origin of these problems is not accurately known. According to a legend, Athens was afflicted with a pestilence and sent to the oracle of Apollo on the Island of Delos for a remedy. Apollo ordered, through his oracle, that the size of his altar at Athens be doubled. The altar was in the form of a cube, and the Athenians immediately built a new cubical altar with each edge twice the length of the edge of the old one. As this had no effect on the pestilence, they tried placing a new altar of the same shape and size by the side of the old one. This also proved of no avail, so another delegation was sent to Delos. There they were told that the new altar must be an exact model of the old one, but of twice the volume. The Athenians then turned the problem over to the geometers, who found themselves unable to solve it. On account of this legend, the problem of the duplication of the cube has come to be known as the "Delian Problem."

Another legend concerning the same problem states that the Egyptian king Menos rejected the plans for a tomb for his son, Glaucus, because it was not large enough and ordered it to be doubled in size but the same form retained. As the tomb was in the shape of a cube, the same problem arose. Many geometers worked on this problem; Hippocrates of Chios (about 430 B.C.) reduced it to one of plane geometry by proving that it is equivalent to finding two mean proportionals between one line and another twice as long. In the proportion $a:x = x:y = y:2a$, $x^2 = ay$, and $y^2 = 2ax$, therefore $x^4 = a^2y^2$, and $x^4 = 2a^3x$ or $x^3 = 2a^3$. This problem, however, is impossible of solution with straightedge and compasses, like the original one.

Archytas of Tarentum (428–347 B.C.) found a curve by using half cylinders which enabled him to find two mean proportionals between two straight lines, but his construction was rejected as being mechanical rather than geometrical.

The trisection of the right angle was known to the Pythagoreans probably as early as 500 B.C., but the general problem of trisecting any angle baffled all attempts. Only in recent years has it been shown that it is impossible of solution with straightedge and compasses.

Hippias of Elis (about 420 B.C.) discovered a transcendental curve by means of which he could divide any angle into any number of equal parts. He also invented a machine for its construction, but Plato refused to allow such methods on the ground that they would widen the limitations of geometry and thus destroy its value. This opinion has been supported by all later geometers.

Pappus (about 200 B.C.), a teacher at Alexandria, offered several solutions of this problem by using non-Euclidean methods, but they were rejected as being outside the limits of geometry.

Nicomedes (about 180 B.C.) invented a curve, the conchoid, and a machine for describing it, by means of which a mechanical solution of the trisection problem was accomplished.

Diocles, a contemporary of Nicomedes, also invented a curve, the cissoid, which furnished other mechanical solutions. These furnished no help in finding a solution to the problem by means of the instruments of plane geometry alone.

Of these three problems, the quadrature of the circle has received the greatest amount of attention. Anaxagoras of Chazomen (500–428 B.C.) is said to have been the first to attempt to construct a square equivalent to a circle. While in prison he studied this problem and wrote a treatise on it.

Antiphon of Athens (480–411 B.C.) used the process of exhaustion in attempting to square the circle. His reasoning, that a straight line segment when sufficiently small coincides with an arc, was defective and was condemned by geometers.

Bryson of Heraclea, a contemporary of Antiphon, used circumscribed as well as inscribed regular polygons and made the false statement that the circle was the arithmetical mean between the perimeters of the circumscribed and the inscribed regular polygons of the same number of sides.

Ahmes found the area of a circle by squaring eight-ninths of the diameter, or by using 3.1604 as the ratio of the circle to its diameter.

Hippocrates (about 470 B.C.) wrote the first textbook on geometry and with Antiphon was the first to use inscribed regular polygons to find the area of a circle.

Ptolemy (87–165 A.D.) used $3\frac{17}{120}$ or 3.1416+ as the value of π and about 500 A.D. a Hindu named Aryabhata, using the same method Archimedes had used, obtained 3.1416+ as the value of π.

Methuis of Holland (1527–1607 A.D.) gave the fraction 355/133 as the value of π, which correct to six decimal places.

Ludolph von Ceulen of Leyden (1527–1616) computed the value of π to thirty-five decimal places. $\pi = 3.14,159,$-

265,358,979,323,846,264,338,327,950,288.[1] This result was considered so important in Germany that there π became known as the "Ludolphian number."

The value of π was computed by Vega to 140 decimal places; by Richter to 509 places; and by Shanks to 707 decimal places.

Lambert proved in 1760 that π is an irrational number and in 1882 Lindemann proved that it is a transcendental number, from which it follows that the quadrature of the circle is impossible with straightedge and compasses.

---

[1]Rupert, W. W., *Famous Geometrical Theorems and Problems*, p. 98.

BROWN UNIVERSITY MATHEMATICAL PRIZES FOR FRESHMEN

> Out of gratitude and respect to his Alma Mater, an alumnus of Brown University has established a fund known as the Brown University Mathematical Prize Fund, from the interest of which prizes are awarded annually by the staff of the Department of Pure Mathematics on the basis of competitive examinations.—*The University of Texas Catalogue.*

The donor provides for four prizes, one of which is offered to students of the calculus, while the other three are given for excellence in high-school mathematics.

> These prizes are offered to the regular freshmen making the best grades on a special voluntary examination to be held on the afternoon of the second Saturday in October. The examination will cover the minimum entrance requirements in mathematics, namely, elementary algebra and plane geometry.—*The University of Texas Catalogue.*

The Brown Prize Examination on high-school algebra and geometry for the session of 1930–31 was held October 11, at 2 P.M., in room 172, Main Building. The questions were as follows:

1. The base of an isosceles triangle is 6 inches and the diameter of the circumscribed circle is 10 inches. Find the length of the sides of the triangle.
2. Given A and B on the same side of the line C D. Draw from A and B two lines meeting C D at the same point and making equal angles with C D.
3. Given a positive and $\neq 1$, prove

$$a + 1/a > 2$$

4. Find x given $(3^x)^x = \dfrac{1}{3^{-4x+8}}$

Pledge.

The winners of the first, second, and third prizes were, respectively, Milo Weaver, of Kirbyville, Texas, awarded \$13.50; Francis A. Hale, of Chester, West Virginia, awarded

\$9.00; and George T. Moore, Jr., of Coleman, Texas, awarded \$4.50.

The second and third prize papers were of almost equal value, Mr. Hale's paper being superior in form and arrangement of proof. Sixty-four students came to take the examination, and forty-seven of them submitted papers. The entire list of grades is appended.

1	85
1	77
1	75
2	65
1	60
9	50
1	45
5	40
4	35
13	25
3	20
6	0

The number attempting the examination is more than double the number in 1928, while the grades on which the awards were made are the highest in a number of years.

THE UNIVERSITY OF TEXAS

FRESHMAN TESTS ON HIGH-SCHOOL ALGEBRA AND GEOMETRY

For the last two years tests on high-school algebra and geometry have been given to the freshman classes in Mathematics 301 and Mathematics 302. The papers have been graded by the various instructors, and the results compiled and sent to the high schools concerned.

In October of this year tests were again given with the results as follows:

Number of schools	361
Number of students	910
Number of students passing	261
Number of students failing	649
Per cent of students passing	28.68

By comparison with the results for the two preceding years, we see that the percentage of those making 60 or more on this year's examination is less than either 1928 or 1929. However, there are several encouraging considerations. Had 50 been the passing grade 43.8 per cent of the students would have passed. Moreover, in several schools where only a small per cent of the students made 60 or more, the average for the entire group from the school was about 55. This average was the result of the large number of grades between 80 and 100.

The list of those who made 90 or more is appended.

Amarillo:
  Alex Mood, 95.
Austin:
  Mary Lucy Dodson, 100.
  Margaret Ezelle, 97.
  Dayton Rutledge, 97.
  Frances Eaves, 96.
  Lillian Ammann, 95.
  Elizabeth Correll, 95.
  Virginia Penick, 95.
  Willie Mae Todner, 92.
  Margaret Knippa, 90.
  Jane Pearce, 90.
Bellville:
  Viola May Dittert, 96.
Cameron:
  Frances Sharpe, 90.
Carrizo Springs:
  Milton Stern, 90.
Chester, W. Va.:
  Francis Hale, 94.
Corpus Christi:
  Hal Rachal, 90.
Corsicana:
  Helen Elizabeth Blackburn, 90.
  Elizabeth Kerr, 90.
  Elizabeth Willie, 90.

Dallas, Forrest Avenue:
Joshua Kahn, 100.
Detroit, Michigan:
Milton Singer, 100.
East Orange, N. J.:
Robert C. Remby, 94.
El Paso:
Dan P. Stewart, 95.
Fort Worth, Central:
John H. Durston, 99.
John H. Durston, 92.
Fredericksburg:
Gifford White, 90.
Galveston:
Fred M. Arend, 97.
Gonzales:
John Romberg, 95.
Houston, San Jacinto:
Frank Merrill, 100.
Harry V. Baker, Jr., 93.
Laredo:
Norman S. Davis, 94.
Mission:
Harold Ross, 94.
Orange:
Mildred Clough, 90.
Pecos:
Mildred Ruhlen, 90.
San Antonio, Alamo Heights:
Robert LaPrelle, 90.
San Antonio, Main Avenue:
Rosalie Robinson, 95.
Smithville:
Thelma Insall, 90.
Stephenville:
Clytie Savage, 95.
Washington, D. C.:
Mary E. Anderson, 100.
Yoakum:
Elizabeth Wimberley, 95.

As is to be expected, the students in Mathematics 302, that is, the students who have had trigonometry in addition to algebra and geometry, average more than 10 points above the students in Mathematics 301, who entered the University with credits in only algebra and geometry.

Special attention is called to the records made by Beaumont, Dallas, Ball High School in Galveston, El Paso, and Graham. The record of schools out of the state is about 32 per cent passing. It is interesting to note that Manila (P.I.), Cardenas (Cuba), McKinley (Honolulu), and Mexico City have each one student who took the tests, and that in each instance the student passed.

The following figures may also be of interest:

There are 138 students making 50– 59.
There are 105 students making 60– 69.
There are 62 students making 70– 79.
There are 54 students making 80– 89.
There are 40 students making 90–100.

MARY E. DECHERD,
For the Department
of Pure Mathematics.

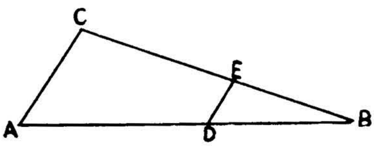
Following are four of the tests used.

1. $1/2 + 3/7 - 2/5 = ?$
2. Reduce 2/9 to a decimal (3 places).
3. Factor completely:
   (a) $x^4 - 16y^8$; (b) $3x - y - 3xz + yz$.
4. Solve the equations

   (a) $\frac{a}{c}x + d = bx + a$.

   (b) $2x^2 - 3x + 1 = 0$.

5. The width of a room is 3/4 of its length. If the width were 2 feet more and the length 2 feet less, the room would be square. Find its dimensions.



6. In the triangle ABC, DE is parallel to AC, and AD = 8, DB = 6, BE = 3. Find BC.

7. Simplify $$\frac{\dfrac{a}{a-b} + \dfrac{a}{a+b}}{\dfrac{b}{a-b} - \dfrac{a}{a+b}}$$

8. Rationalize the denominator of $$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

1. Find the value of $3/5 + 5/2 - 4/7$.
2. Divide 0.02683 by 0.007213 to two decimal places.
3. Simplify: $\dfrac{2+\sqrt{8}}{\sqrt{2}}$; (b) $\dfrac{x - 1/x}{x+1}$.
4. Factor completely:

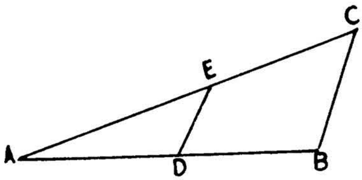
   (a) $y^4 - 81$; (b) $a^2b + 3a^2 + ab + 3a$.

5. Find two numbers whose sum is 11 and whose difference is 29.
6. Find altitude, dropped on side 5, in the $\triangle$ whose sides are 3, 4, 5.
7. Solve: (a) $2x^2 - x - 6 = 0$;
   (b) $3/x + 2 = 1/x - 2$.
8. In $\triangle$ ABC draw DE so that AD/DC = BE/EC with DC/AD = 3/7.

1. Evaluate $(2\tfrac{2}{3} + 5/6)\,(2/7)^{-1}$.
2. Find $\sqrt{7}$ to two places of decimals.
3. Factor completely:
   (a) $81y^8 - 16x^4$; (b) $a^2 - b^2 + 2a + 2b$.
4. Simplify: $\dfrac{\sqrt{12} + \sqrt{2}}{\sqrt{27}}$.
5. Simplify: $\dfrac{a^2 - ab + b^2}{a - b} \times \dfrac{a^2 - b^2}{a^3 + b^3}$.
6. Solve for x and check value or values found:
   (a) $\dfrac{4x - 2}{5} + \dfrac{5}{8}x = \dfrac{3x}{4} + 5$;
   (b) $2x^2 - 7x + 5 = 0$.
7. Find the radius of a circle whose area is 78.54 sq. in.
8. Given a triangle of sides 5, 12, 13 in length. Find the length of the altitude upon the side of length 13 of this triangle.

1. $2/3 + 3/5 - 5/7 = ?$
2. Reduce 12/17 to a decimal (3 places).
3. Factor completely:
   (a) $16x^4 - 81y^8$; (b) $2a + 2b + a^2 - b^2$.
4. Simplify: (a) $\dfrac{\sqrt{3} + \sqrt{12}}{2\sqrt{27}}$; (b) $\dfrac{x^2 - 1}{x - 1/x}$.

5. Solve the equations:
   (a) $3/x + 2 = 1/x - 2$;
   (b) $6x^2 + 7x - 5 = 0$.
6. A rectangular field of area 1,500 square feet is surrounded by a fence whose total length is 170 feet. Find the length and width of the field.



7. In the triangle ABC, DE is parallel to BC, and AB = 8, AC = 12, AD = 5. Find EC.

8. If the hypotenuse of a right triangle is 26 and one side is 10, find the other side.

	Total	Passed	Failed
Abilene	6	1	5
Albany	2	0	2
Alief	1	0	1
Alliance Acad., Cambridge Springs, Pa.	1	1	0
Alvin	2	0	2
Alice	1	0	1
Alvarado	1	1	0
A. & M. Consolidated	1	0	1
Amarillo	6	1	5
Anadarko	1	0	1
Anson	6	0	6
Thos. Arnold	1	1	0
Athens	4	1	3
Austin	136	55	81
Bainbridge (Ga.)	1	0	1
Baird	1	0	1
Barston (Kansas City)	1	0	1
Bartlett	3	0	3
Bastrop	4	0	4
Bay City	1	0	1
Beaumont	7	5	2
Beeville	7	0	7
Bellevue	1	1	0
Bellville	6	2	4
Belton	1	0	1
Benhie	1	0	1
Big Spring	1	0	1
Bishop	1	0	1
Brady	1	0	1
Brenham	1	0	1
Bryan	1	0	1
Bolton	1	0	1
Britton, S. Dak.	1	0	1
Bruceville Eddy	1	0	1
Burkburnett	1	0	1
C. E. Byrd (Shreveport)	3	0	3
Caldwell	4	0	4
Calvert	1	1	0
Carrizo Springs	1	1	0
Castle Hts. Mil. Acad.	1	1	0
Celina	1	0	1
Chester (W. Va.)	1	1	0
Cheyenne (Okla.)	1	0	1
Cheyenne (Colorado Springs, Colo.)	1	1	0
Chico	1	0	1
Chilton	1	1	0
Cisco	3	1	2
Clarksville	3	0	3
Cleburne	2	0	2
Clifton Col. Acad.	1	0	1
Clovis (N. Mex.)	1	0	1
Coleman	6	1	5
Colombia (S. C.)	1	1	0
Columbus	1	1	0
Comanche	2	0	2
Cooper	1	0	1
Corpus Christi	9	1	8
Corsicana	8	3	5
Cripple Creek (Colo.)	1	0	1
Crockett	2	0	2
Cuero	2	0	2
Daingerfield	2	0	2
Bryan St. (Dallas)	2	1	1
Forrest Ave. (Dallas)	10	6	4
Highland Park (Dallas)	7	4	3

	Total	Passed	Failed
North Dallas (Dallas)	2	2	0
Oak Cliff (Dallas)	5	2	3
Sunset (Dallas)	2	0	2
Woodrow Wilson (Dallas)	6	3	3
West Dallas (Dallas)	1	0	1
Decatur (Ill.)	1	0	1
Decatur	1	0	1
De La Salle College (Manila, P. I.)	1	1	0
Del Rio	2	1	1
Denison	2	0	2
Denton	1	0	1
Detroit Central	3	1	2
Detroit Northern	2	1	1
Deweyville	1	0	1
Dormont (Pittsburg, Pa.)	1	1	0
Eagle Lake	1	0	1
Eagle Pass	3	2	1
Eastland	1	0	1
East Orange (N.J.)	2	2	0
E. T. S. T. C.	2	1	1
Edinburg	1	0	1
Edna	2	1	1
El Campo	4	0	4
Elgin	1	1	0
Eldorado	1	0	1
El Paso	7	3	4
Ennis	3	1	2
Falfurrias	1	0	1
Farmersville	2	0	2
Florence	1	0	1
Floresville	3	0	3
Floydada	2	0	2
Follett	1	1	0
Foraker (Okla.)	1	1	0
Foreman (Ark.)	1	0	1
Ft. Collins (Colo.)	1	0	1
Ft. Stockton	5	1	4
Central (Ft. Worth)	15	5	10
North Side (Ft. Worth)	1	0	1
Polytechnic (Ft. Worth)	4	0	4
Franklin	2	2	0
Frederick (Okla.)	1	0	1
Fredericksburg	4	1	3
Freeport	3	2	1
Gainesville	1	0	1
Ball High (Galveston)	20	9	11
George School, Pa.	1	1	0
Giddings	1	0	1
Gilmer	3	1	2
Gonzales	5	2	3
Goldthwaite	1	0	1
Goose Creek	2	1	1
Gorman	2	1	1
Grand Prairie	1	0	1
Greenville	1	0	1
Groesbeck	1	0	1
Graham	6	3	3
Granger	4	1	3
Groom	2	0	2
Groveton	1	0	1
Hamilton	1	0	1
Harlandale	2	1	1
Harlingen	2	0	2
Harrison, Ark.	1	1	0
Hearne	1	0	1
Hempstead	2	1	1
Henderson	3	2	1
Henrietta	1	1	0
Hereford	4	0	4
High Bridge, N.J.	1	0	1
Hillsboro	2	0	2
Hollis	1	1	0
Hondo	1	0	1
Honey Grove	1	0	1
Hot Springs, Ark.	1	1	0
Central (Houston)	2	0	2
Heights (Houston)	1	1	0
Jeff Davis (Houston)	3	2	1
Reagan (Houston)	1	1	0
Sam Houston (Houston)	9	4	5
San Jacinto (Houston)	19	5	14
Hubbard	1	0	1
Hull-Daisetta	1	0	1
Humble	1	0	1
Huntsville	1	0	1
Hutto	4	1	3
Jacksboro (1904)	1	0	1
Jackson, Miss.	1	0	1
Andrew Jackson (Jacksonville, Fla.)	1	1	0
Jacksonville	1	1	0
Jasper, Mo.	1	0	1
Jellico	1	0	1
Johnstown, Pa.	1	0	1
Jonestown, Miss.	1	0	1
Joplin, Mo.	1	0	1
Junction	2	0	2
Katy	1	0	1
Kaufman	1	0	1
Kenedy	1	1	0
Killeen	1	0	1
Kirby Woodville, Tex.	1	1	0
Kirbyville	4	0	4
Kountze	1	0	1

	Total	Passed	Failed
Lake Forest Acad.	1	1	0
Lamesa	1	1	0
Lampasas	3	0	3
Lancaster	1	0	1
La Progresiva (Cardenas, Cuba)	1	1	0
Laredo	6	3	3
Lawrence (Ala.)	1	0	1
League City	1	1	0
Liberty	3	0	3
Llano	4	2	2
Lockhart	4	1	3
Longview	1	1	0
Lowell (San Francisco)	1	0	1
Lufkin	3	0	3
Luling	1	0	1
Lyford	3	1	2
Mansfield (La.)	2	0	2
Manuel Ojinogs (Mex.)	1	0	1
Marlin	6	1	5
Marshall	3	1	2
Masonic Home	1	1	0
Mathis	1	0	1
McAllen	3	0	3
McCallie (Chattanooga, Tenn.)	1	1	0
McGregor	2	0	2
McKinley (Washington, D.C.)	1	1	0
McKinley (Honolulu)	1	1	0
McKinney	3	0	3
Memphis	1	0	1
Mercedes	4	1	3
Meridian	3	1	2
Mexia	2	0	2
Mexico City (American)	1	1	0
Miami (Fla.)	1	1	0
Miami	1	0	1
Midland	2	0	2
Milby	1	0	1
Mills (N.M.)	1	0	1
Mineral Wells	3	1	2
Mission	3	2	1
Montezuma	1	0	1
Montgomery (Ala.)	1	1	0
Monthalia	1	0	1
Monticello (Miss.)	1	0	1
Moody	1	0	1
Mt. Pleasant	2	0	2
Munday	1	1	0
Mupel	1	0	1
Muskogee (Okla.)	1	0	1
Nacogdoches	1	0	1
Nogales (Ariz.)	1	0	1
Navasota	1	0	1
Nevada	1	0	1
New Braunfels	1	0	1
New Philadelphia	1	0	1
Nicholas Senn (Chicago)	1	0	1
Nixon	2	0	2
N. M. Mil. Inst.	1	1	0
Oak Park (Ill.)	2	0	2
Odessa	1	0	1
Central (Okla. City)	1	0	1
Orange	3	1	2
Orchard	1	0	1
Our Lady of the Lake	1	0	1
Overton	1	0	1
Palacios	1	0	1
Palestine	4	2	2
Panhandle	2	0	2
Paris	2	0	2
Payne	1	0	1
Peace Inst. (Raleigh, N. C.)	1	1	0
Pearsall	3	1	2
Pecos	4	1	3
Petrolia	1	0	1
Pharr	1	0	1
Philadelphia (Miss.)	1	0	1
Pilot Point	1	0	1
Pinckneyville (Ill.)	1	0	1
Pineville	1	0	(*)
Plainview	1	0	1
Port Arthur	4	0	4
Port Lavaca	3	1	2
Post (55)	1	0	1
Poteet	1	0	1
Quanah	2	0	2
Ralls	1	0	1
Rankin	3	0	3
Raymondville	2	0	2
Richmond	1	0	1
Rising Star	1	1	0
Roaring Springs	1	0	1
Robstown	1	0	1
Rockdale	1	1	0
Rock Island (Ill.)	3	1	2
Rocksprings	1	0	1
Rockwall	1	0	1
Rogers	1	1	0
Roosevelt (Des Moines, Ia.)	1	0	1
Rosenberg	1	1	0

*No paper.

	Total	Passed	Failed
Roswell (N. Mex.)	1	0	1
Round Rock	1	0	1
Runge	4	1	3
Rush Springs (Okla.)	1	0	1
San Angelo	1	1	0
Alamo Heights (San Antonio)	4	1	3
Brackenridge (San Antonio)	6	1	5
Main Avenue (San Antonio)	26	7	19
San Augustine	1	0	1
San Benito	2	0	2
San Gabriel	1	0	1
San Germain, P. R. Polytech Hi	1	1	0
San Jacinto	1	0	1
S. M. B. A. Camp Marlamont	1	0	1
San Marcos	2	0	2
San Saba	1	0	1
Saratoga	2	2	0
Schreiner	1	0	1
Schreiner Inst.	4	0	4
Scoville (N.Y.C.)	1	1	0
Seguin	2	1	1
Lutheran Acad. (Seguin)	1	0	1
Seward Park (N.Y. C.)	1	0	1
Seymour	1	0	1
Sherman	1	0	1
Shine	1	0	1
Shiner	2	0	2
Shreiner	1	0	1
Smithville	3	1	2
Somerville	1	0	1
Sour Lake	2	1	1
S. W. T. S. T. C. Prep.	1	0	1
S. P. N. S. (Wis.)	1	1	0
Stamford	2	0	2
State Home High (Corsicana)	2	1	1
Stephenville	1	1	0
St. Benedicts (Atchison, Kans.)	1	0	1
St. Chas. (Mo.)	1	0	1
St. Josephs (Victoria)	1	1	0
St. Louis University High	1	0	1
St. Louis College (Honolulu)	1	0	1
St. Mary's Academy (Austin)	3	0	3
St. Thomas College (Houston)	1	0	1
St. Xaviers (Cincinnati, Ohio)	1	0	1
W. C. Stripling High	1	0	1
Sulphur Springs	1	0	1
Central (Syracuse, N. Y.)	1	0	1
Taft	2	1	1
Taylor	9	0	9
Teague	1	0	1
Temple	2	0	2
Terrill Prep. (Dallas)	3	0	3
Thorndale	2	1	1
Thornton	2	2	0
Tivy (Kerrville)	1	0	1
T. M. I. (San Antonio)	4	0	4
Topeka (Kans.)	1	0	1
Troy	1	0	1
Troup	1	0	1
Tulia	3	0	3
Tyler	2	1	1
Urbana (Ill.)	1	0	1
Urseline Academy (Dallas)	1	1	0
Uvalde	1	0	1
Valley Mills	1	0	1
Vernon	2	0	2
Villia Maria, Bryan (59)	1	0	1
Waco	12	2	10
Ward Belmont (Nashville, Tenn.)	1	0	1
Western (Washington, D. C.)	1	1	0
Weimar	1	0	1
Wellington	1	0	1
Weslaco (58)	1	0	1
Wesley College (Greenville)	1	1	0
Wesleyan Inst. (San Antonio)	1	0	1
West	3	0	3
West Columbia	1	0	1
West Dallas	1	0	1
Westmoorland College	1	0	1
W. T. S. T. C. High	1	0	1
Wharton	3	0	3
Whitesboro (N. Y.)	1	0	1
Wichita Falls	12	3	9
Wilkinson (Ind.)	1	1	0

	Total	Passed	Failed
Wills Point	1	1	0
Winnsboro (56)	1	0	1
Wolvin (Texas City)	1	0	1
Woodville	1	1	0

	Total	Passed	Failed
Woodstock Community (Ill.)	1	0	1
Yazoo City (Miss.)	1	0	1
Yoakum	6	1	5
Yoe (Cameron)	7	3	4
Yorktown	3	1	2